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THREE ESSAYS ON OLIGOPOLY AND FINANCIAL STRUCTURE

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THREE ESSAYS ON OLIGOPOLY AND
FINANCIAL STRUCTURE

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Dedicated to Kyung Eun, Hannah and my parents
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THREE ESSAYS ON OLIGOPOLY AND FINANCIAL STRUCTURE

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This dissertation examines interactions between financial structure and production equilibrium of repeated duopoly games. In repeated games, there emerges the strategic bankruptcy effect (predation threat) in addition to the well-known limited liability effect. In the first essay, I find that when the limited liability effect dominates the strategic bankruptcy effect, debt commits the issuing firm to choose an aggressive output level and reduces its rival’s output level so that by backward induction the optimal debt level is strictly positive. This occurs when the discount factor is very low. When the discount factor is zero, firms choose exactly the same debt level as in Brander and Lewis (1986, AER). On the other hand, when the strategic bankruptcy effect dominates the limited liability effect, debt reduces the issuing firm’s output level but induces an increase in its rival’s output level so that non-debt financing is the optimal financial decision, as it is in the long purse theory.
In the second essay, in the case where firms’ demand shocks are perfectly correlated, the predation threat cannot occur in a symmetric debt-output equilibrium, because no firm has a chance to be a monopolist in the next period. In this setting, I find that there is no symmetric pure strategy equilibrium but there exist two asymmetric pure output strategy equilibria and one symmetric mixed output equilibrium. Sequentially I find that for any discount factor, there exists an optimal debt level which is supported by firms. The two asymmetric pure output equilibria are used as a punishment against the firm deviating from the optimal debt level in the mixed strategy equilibrium; for the deviating firm the asymmetric output equilibrium is worse than the mixed output strategy equilibrium. In the repeated game, this punishment supports a symmetric optimal debt level and the mixed output strategy.

In the third essay, I show that the limited liability effect also appears in the risk averse manager case. However, the relationship between the degree of risk aversion and the limited liability effect is theoretically ambiguous. Simulation results show that this relationship depends on the degree of risk aversion
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Chapter 1

Oligopoly and Financial Structure in the Multi-Stage Games

1.1 Introduction

Contrary to existing theories, recent empirical work by Chevalier (1995), Kovenock and Phillips (1995, 1997), and particularly Phillips (1995) shows that in concentrated industries, high leverage tends to be correlated with low output, high prices, and more passive investment behavior; that is, debt seems to have anti-competitive effects on product market. However, the existing theories of Brander and Lewis (1986), Maksimovic (1988), and Stenbacka (1994) argue that debt commits a leveraged firm to a more aggressive output strategy; that is, debt financing has a positive effect on production level.

In this paper, I examine interactions between financial structure and production equilibrium of a repeated duopoly game. I present how the debt financing affects the firm’s output decision, and by backward induction how the optimal debt-equity financing decision is determined. This research proposes a theoretical explanation for the empirical evidence. Building on Brander and Lewis’ (1986) model of interactions between financial decision and production strategy, I show that the contradiction between theory and evidence
has emerged because existing theories ignore the predation threat in repeated games.

There are two main ways in which the financial structure can affect output markets. The first one is the limited liability effect of debt financing. Leveraged firms have an incentive to pursue output strategies that raise returns in good states and lower returns in bad states. The basic point is that shareholders ignore reductions in returns in bankruptcy states, since bondholders become the residual claimants. As debt levels change, the distribution of returns to shareholders over the different states changes, which in turn changes the output strategy favored by shareholders. Therefore, a highly leveraged firm concentrating on good states selects a more aggressive production level, and it commits an unleveraged rival to a passive production level. The existing theories contribute to the research on the limited liability effect of debt financing, and the seminal work by Brander and Lewis (1986) shows that firms hold positive debt levels endogenously as an equilibrium strategy.

The second linkage between output and the financial market is the strategic bankruptcy effect. A firm’s susceptibility to financial distress depends on its financial structure, and its fortunes will usually improve if one or more of its rivals can be driven into financial distress. Therefore, firms might make output market decisions that raise the chances of driving their rivals into insolvency. Since the possibility of financial distress for each firm is contingent on its financial structure, a highly leveraged firm is more vulnerable relative to an unleveraged firm. Therefore the strategic bankruptcy effect implies that,
in a repeated oligopoly game, firms select non-debt financing in order to avoid predation by rivals or prey upon rivals. The long purse theory has contributed to the research on the strategic bankruptcy effect of debt financing; Fudenberg and Tirole (1986) explain that an entrant will decide to stay or exit after entering the market and uses its current profit to make decisions because it is uncertain of its future profitability. The established firm will take competitive actions to lower the entrant’s realized profits so that the entrant will more likely exit. Poitevin (1989) illustrates that financiers are uncertain of the entrant’s true value but that they know the incumbent’s true worth so that the entrant must issue debt to signal his quality to investors. Consequently, the high-value entrant comes into the market heavily leveraged compared to the incumbent, so the incumbent has an incentive to prey upon the new entrant.

In this paper I focus on how these two effects of debt financing on the output strategies work in a repeated oligopoly game. The analysis I offer here illustrates that the output equilibrium is determined by the limited liability effect and the strategic bankruptcy effect in the output market so that, by backward induction, the two effects determine the optimal financial decision. Existing theories of the limited liability effect do not include the strategic bankruptcy effect of repeated oligopoly games. Brander and Lewis (1986) do not have to consider predation threat because they set a static game in the production market. However, Maksimovic (1988) and Stenbacka (1994) do not explain why leveraged firms are not vulnerable to predation in their infinitely repeated oligopoly games.
On the other hand, the long purse theory cannot provide any endogenous reasons for holding debt; it only offers exogenous conditions such as liquidity constraint. Fudenberg and Tirole (1986) explain that an entrant has a financing requirement larger than the incumbent and thus must issue more debt. Poitevin (1989) shows that since a new entrant has no past track record, its financial decisions are constrained by investors. Therefore, the long purse theory only works in asymmetric conditions and considered skeptically when dealing with symmetric cases. Bolton and Scharfstein (1990) analyze the financial contract which optimizes the trade-off relationship between moral hazard (limited liability) and predation threat. However, they focus on how the optimal financial covenants determined by the two effects, but do not explain how output decision is affected directly. Furthermore, they assume an exogenously asymmetric environment where one firm needs to borrow money from investors while the rival does not. Their work implicitly excludes any endogenous or symmetric equilibrium.

This paper is an extension of Brander and Lewis (1986), and the model is a two-stage sequential duopoly game as it is in their model. In stage 1, the two firms decide upon financial structure. In stage 2, they select output levels taking as given the financial composition determined in stage 1. Although the Brander and Lewis model has a static production game in stage 2, this model has the two firms competing in repeated production games. When adding one more static game into stage 2, the predation threat by the rival should be taken into account. This paper specially examines the effects on the first
period output equilibrium by the strategic bankruptcy effect in addition to the limited liability effect in the repeated game. The equilibrium concept used in this study is the sequentially rational Nash equilibrium.

The output decisions of firms are determined before a random shock in demand is revealed. Once profits are determined, the leveraged firm has the duty to pay its debt payment to creditors every period. When the firm’s profits are insufficient to meet the amount it must pay, the firm with debt must go bankrupt. Therefore, if one firm issues debt, then the rival firm has an incentive to prey upon this leveraged firm in the first-period output market game, in order to monopolize the next period’s market. Hence the leveraged firm is very vulnerable in the first period due to the strategic bankruptcy effect. On the other hand, shareholders of the leveraged firm have the limited liability, so they lose not whole profits but rather stock returns, when the firm goes bankrupt. The firm with high debt has an incentive to choose an aggressive output strategy relative to the unleveraged firm. Therefore, when the strategic bankruptcy effect dominates the limited liability effect, increases in debt reduce the firm’s output level so that the firm can raise its survival rate and induce its rival to raise its output for predation purpose. The rival does this so that, by backward induction, finance entirely with equity is optimal. When the limited liability effect dominates the strategic bankruptcy effect, increases in debt induce reverse results so that, by backward induction, the optimal debt level is strictly positive.

The outline of the paper is as follows. Section 2 sets out the basic
model. Section 3 is devoted to the output market equilibrium and shows the
dependence of output equilibrium on financial structure. Section 4 examines
the selection of debt levels and describes how output market considerations
influence financial structure. Section 5 contains simulation results under some
restricted conditions. The last section summarizes results and provides con-
clusions.

1.2 The Model

The model is an extension of Brander and Lewis’ (1986) wherein two
identical firms compete in a one period output market with a fixed financial
structure. To this model, I add one period in the output market so that the
two firms compete in a twice-repeated production game with a given financial
structure. The financial decision formula is the same as in the Brander and
Lewis model (BL model, hereinafter).

The model consists of two stages, a financial decision and a repeated
output game. Each firm needs $I$ as the investment for a business, and it must
decide whether to issue debt or stock to finance $I$. The financial structure
of each firm is defined by $d_i$, which is the amount of debt obligation firm $i$
has to pay each period. Firm $i$ and creditors contract a debt covenant, which
requires that firm $i$ borrow $W_i$ from creditors and has to repay $d_i$ to creditors
each period. The debt level is assumed to be chosen before output decisions.

If firm $i$ is unable to meet its debt obligation in a certain period, firm
i goes bankrupt, and its bondholders receive residual profits. If firm i goes
bankrupt in the first period, it cannot play in the second period output market.
The game tree is shown in Figure 1.1.

Firms i and j play a repeated game in an output market where they
produce competing products \( q_{it} \) and \( q_{jt} \) in each period \( t = 1 \) and 2, respectively.
For concreteness, we assume there exists Cournot quantity competition in
the output market. Profit for firm i at t, which is defined as the difference
between revenue and variable costs, and given by \( \Pi_{it}(q_{it}, q_{jt}, \varepsilon_{it}) \). The random
variable \( \varepsilon_{it} \) reflects the effects of an uncertainty on firm i at the period t. For
simplicity’s sake, it is assumed to be uniformly distributed over the interval
[0, 1]. Additionally, I assume that \( \varepsilon_{it} \) and \( \varepsilon_{jt} \) are independent and identically
distributed; that is, \( F(\varepsilon_{it}, \varepsilon_{jt}) = F(\varepsilon_{it}) F(\varepsilon_{jt}) = \varepsilon_{it} \cdot \varepsilon_{jt} \). Each periods’ random
variables, \( \varepsilon_{it} \) and \( \varepsilon_{jt} \), are revealed after production decisions, \( q_{it} \) and \( q_{jt} \), are
made respectively.

I assume that \( \Pi_{it} \) satisfies the usual properties:

\[
\frac{\partial^2 \Pi_{it}}{\partial q_{it}^2} < 0, \quad \frac{\partial \Pi_{it}}{\partial q_{jt}} < 0, \quad \frac{\partial^2 \Pi_{it}}{\partial q_{it} \partial q_{jt}} < 0
\]  

(1.2.1)

I adopt the convention that higher values of \( \varepsilon_{it} \) lead to higher operating profits,
meaning that higher realizations of \( \varepsilon_{it} \) correspond to better states of the world.

\[
\frac{\partial \Pi_{it}(q_{it}, q_{jt}, \varepsilon_{it})}{\partial \varepsilon_{it}} > 0
\]  

(1.2.2)

Since the two firms play in a standard quantity competition, I additionally
assume that higher values of \( \varepsilon_{it} \) correspond to upward shifts in the marginal
revenue schedule facing the firms\(^1\):

\[
\frac{\partial^2 \Pi_{it}(q_{it}, q_{jt}, \varepsilon_{it})}{\partial q_{it} \partial \varepsilon_{it}} > 0, \quad \frac{\partial^2 \Pi_{it}(q_{it}, q_{jt}, \varepsilon_{it})}{\partial q_{jt} \partial \varepsilon_{it}} \leq 0
\] (1.2.3)

If only one of the two firms survives and the other goes bankrupt in the first period \((t = 1)\), then the firm \(i\) exists as a monopolist in the second-period.

\(^1\)Brander and Lewis (1986) explain that this assumption is the standard case under quantity competition, while \(\frac{\partial^2 \Pi_{it}(q_{it}, q_{jt}, \varepsilon_{it})}{\partial q_{it} \partial \varepsilon_{it}} < 0\) may arise when firms engage in other forms of competition besides quantity or price competition.
market \((t = 2)\). Then the monopolist \(i\) decides the monopoly output level, \(q^M_{i2}\), and receives the monopoly profits, \(\Pi^M_{i2}(q^M_{i2}, \varepsilon_{i2})\), in the second period. For simplicity’s sake, I assume that monopoly profits are sufficiently high: that is, \(d_i < \Pi^M_{i2}(q^M_{i2}, \varepsilon_{i2})\) for all \(\varepsilon_{i2} \in [0, 1]\). Then the expected stock return of the monopolist at \(t = 2\) is as follows;

\[
S^M_{i2} = \int_0^1 \left[ \Pi^M_{i2}(q^M_{i2}, \varepsilon_{i2}) - d_i \right] d\varepsilon_{i2} = \int_0^1 \Pi^M_{i2}(q^M_{i2}, \varepsilon_{i2}) d\varepsilon_{i2} - d_i \tag{1.2.4}
\]

Similarly, if the firm \(i\) survives alone in the second period, then this firm chooses a monopoly output level and can pay the debt duty \(d_i\) for any state. Therefore the expected debt returns in the second period, when the firm is a monopoly, is given by

\[
D^M_{i2} = \int_0^1 d_i d\varepsilon_{i2} = d_i \tag{1.2.5}
\]

If both firms \(i\) and \(j\) survive in the first period, then they choose outputs \(q_{i2}\) and \(q_{j2}\) at the second duopoly game like they did in the first period. In the second period, neither firm faces a future stage since the second period is the end of the game, each firm makes decisions as shown in the BL model. Then the expected stock return of the firm \(i\) at the second period duopoly game is as follows;

\[
S^{Du}_{i2} = \int_0^1 \max\{\Pi_{i2}(q_{i2}, q_{j2}, \varepsilon_{i2}) - d_i, 0\} d\varepsilon_{i2} \tag{1.2.6}
\]

\[
= \int_{z_{i2}}^1 \{\Pi_{i2}(q_{i2}, q_{j2}, \varepsilon_{i2}) - d_i\} d\varepsilon_{i2}
\]

where \(z_{i2}\) is defined by

\[
\Pi_{i2}(q_{i2}, q_{j2}, z_{i2}) - d_i = 0, \tag{1.2.7}
\]
assuming $0 < z_{i2} < 1$. When $\varepsilon_{i2} = z_{i2}$, firm $i$ can just meet its debt obligations with nothing left over. Firm $i$ would go bankrupt with probability $z_{i2}$ and be able to repay $d_i$ with $(1 - z_{i2})$ in the second period. If both firms survive in the second period, then both firms play the duopoly game, and the expected returns of debt holders at the second period duopoly game, $D_{i2}^{Duo}$ is given by

$$D_{i2}^{Duo} = \int_0^{z_{i2}} \Pi_{i2}(q_{i2}, q_{j2}, \varepsilon_{i2})d\varepsilon_{i2} + \int_{z_{i2}}^1 d_i d\varepsilon_{i2} \quad (1.2.8)$$

The first period expected stock return of firm $i$ is defined by

$$S_{i1} = \int_0^1 \max\{\Pi_{i1}(q_{i1}, q_{j1}, \varepsilon_{i1}) - d_i, 0\}d\varepsilon_{i1} \quad (1.2.9)$$

$$= \int_{z_{i1}}^1 \{\Pi_{i1}(q_{i1}, q_{j1}, \varepsilon_{i1}) - d_i\}d\varepsilon_{i1}$$

where $z_{i1}$ is defined by

$$\Pi_{i1}(q_{i1}, q_{j1}, z_{i1}) - d_i = 0, \quad (1.2.10)$$

assuming $0 < z_{i1} < 1$. Therefore, firm $i$ survives period 1 with $\Pr(\varepsilon_{i1} > z_{i1}) = 1 - z_{i1}$. In the second period, $t = 2$, it will face duopoly competition with $(1 - z_{i1})(1 - z_{j1})$, and be the monopolist with $(1 - z_{i1})z_{j1}$. If the first period demand shock $\varepsilon_{i1}$ is less than $z_{i1}$, the debt holders receive the residual profits, while they receive the whole $d_i$ if the revealed shock is greater than $z_{i1}$. Therefore, the expected debt return of the first period, $D_{i1}$, is given by

$$D_{i1} = \int_0^{z_{i1}} \Pi_{i1}(q_{i1}, q_{j1}, \varepsilon_{i1})d\varepsilon_{i1} + \int_{z_{i1}}^1 d_i d\varepsilon_{i1} \quad (1.2.11)$$

Given debt levels $(d_i, d_j)$, the firm is assumed to choose output levels with the objective of maximizing the expected present value of the firm to
the shareholders with discount factor, \( \beta \). The natural assumption is that managers maximize the present equity value when debt levels are determined as given. The expected present value of firm \( i \)'s equity holder is the summation of discounted stock returns, as represented by \( V_i \):

\[
V_i(q_{i1}, q_{j1}, q_{i2}, q_{j2}, q_{i2}^M, d_i, d_j) = S_{i1} + \beta (1 - z_{i1}) [z_{j1} S_{i2}^M + (1 - z_{j1}) S_{i2}^D] \\
= \int_{z_{i1}}^{1} \left[ \Pi_{i1}(q_{i1}, q_{j1}, \varepsilon_{i1}) - d_i \right] d\varepsilon_{i1} \\
+ \beta (1 - z_{i1}) (z_{j1}) \int_{0}^{1} \left[ \Pi_{i2}(q_{i2}, q_{j2}, \varepsilon_{i2}) - d_i \right] d\varepsilon_{i2} \\
+ \beta (1 - z_{i1}) (1 - z_{j1}) \int_{z_{i2}}^{1} \left[ \Pi_{i2}(q_{i2}, q_{j2}, \varepsilon_{i2}) - d_i \right] d\varepsilon_{i2}
\]

(1.2.12)

where the discount factor is denoted by \( \beta \in [0, \infty) \).\(^2\)

The expected present value of firm \( i \)'s debt holders is the summation of discounted debt returns, as defined by \( W_i \):

\(^2\)If we normalize two stock returns of two periods with the discount rate \( \delta \), then the sum of stock returns is given by,

\[
\delta S_{i1} + (1 - \delta) \{(1 - z_{i1}) [z_{j1} S_{i2}^M + (1 - z_{j1}) S_{i2}] \}
\]

where \( \delta \in (0, 1] \)

Therefore, the present value of stockholders is the stock return sum divided by the discount rate \( \delta \) and \( \beta \) is equal to \( (1 - \delta) / \delta \) as follows:

\[
V_i = S_{i1} + \frac{1 - \delta}{\delta} \{(1 - z_{i1}) [z_{j1} S_{i2}^M + (1 - z_{j1}) S_{i2}] \}
\]
\[ W_i(q_{i1}, q_{j1}, q_{i2}, q_{j2}, q_{i2}^M, |d_i, d_j) \]

\[ = D_{i1} + \beta(1 - z_{i1})[z_{j1} D_{i2}^M + (1 - z_{j1}) D_{i2}] \]

\[ = \int_0^{z_{i1}} \Pi_{i1}(q_{i1}, q_{j1}, \varepsilon_{i1}) d\varepsilon_{i1} + \int_{z_{i1}}^1 d_i d\varepsilon_{i1} \]

\[ + \beta(1 - z_{i1})(z_{j1}) \int_0^1 d_i d\varepsilon_{i2} \]

\[ + \beta(1 - z_{i1})(1 - z_{j1}) \left[ \int_0^{z_{i2}} \Pi_{i2}(q_{i2}, q_{j2}, \varepsilon_{i2}) d\varepsilon_{i2} + \int_{z_{i2}}^1 d_i d\varepsilon_{i2} \right] \]

In short, firm \( i \) (firm \( j \) ) borrows the creditors’ expected present value, \( W_i \) (\( W_j \)) and has to pay debt obligation \( d_i \) (\( d_j \)) to creditors every period. Firm \( i \) borrows between zero and \( I \) (i.e. \( 0 < W_i < I \))\(^3\). Recognizing \( d_i \) and \( d_j \), both stockholder managers choose the first period output levels, \( q_{i1} \) and \( q_{j1} \), and choose the second period output levels, \( q_{i2}, q_{j2}, q_{i2}^M \) and \( q_{j2}^M \in M \) in order to maximize the expected present value of stock. Note that managers maximize not the total firm value, \( V_i + W_i - I \) but the expected stock value, \( V_i \), because the stockholder managers are interested in only the stock value.

### 1.3 Output Market Equilibrium

In this section I examine how the first-period output equilibrium is different from BL equilibrium and distinguish the predation effect from the limited liability effect. Taking existing debt levels \( d_i \) and \( d_j \) as predetermined,

\(^3\)Note that debt obligation amounts, \( d_i \) and \( d_j \) are called debt levels in this paper.
the management of each firm chooses output levels, \( q_{i1} \), \( q_{i2} \) and \( q_{i2}^M \) to maximize the present value of stock, \( V_i \).

The expression (1.2.10) shows the implicit dependence of \( z_i \) on \( d_i \), \( q_i \) and \( q_j \). It is useful to report the following derivatives:

\[
\frac{dz_{i1}}{dd_i} = \frac{1}{\left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial z_{i1}} \right)} > 0 \quad (1.3.1.1)
\]

\[
\frac{dz_{i1}}{dd_j} = 0 \quad (1.3.1.2)
\]

\[
\frac{dz_{i1}}{dq_{i1}} = -\frac{\left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial q_{i1}} \right)}{\left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial z_{i1}} \right)} \quad (1.3.1.3)
\]

\[
\frac{dz_{i1}}{dq_{j1}} = -\frac{\left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial q_{j1}} \right)}{\left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial z_{i1}} \right)} > 0 \quad (1.3.1.4)
\]

\[
\frac{d^2 z_{i1}}{dq_{i1}dq_{j1}} = -\frac{\left( \frac{\partial^2 \Pi_{i1}(z_{i1})}{\partial q_{i1}\partial q_{j1}} \right)}{\left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial z_{i1}} \right)^2} < 0 \quad (1.3.1.5)
\]

\[
\frac{d^2 z_{i1}}{dq_{i1}dd_i} = -\frac{\left( \frac{\partial^2 \Pi_{i1}(z_{i1})}{\partial q_{i1}\partial d_i} \right)}{\left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial z_{i1}} \right)} < 0 \quad (1.3.1.6)
\]

\[
\frac{d^2 z_{i1}}{dq_{j1}dd_i} = -\frac{\left( \frac{\partial^2 \Pi_{i1}(z_{i1})}{\partial q_{j1}\partial d_i} \right)}{\left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial z_{i1}} \right)} \geq 0 \quad (1.3.1.7)
\]

I adopt Brander and Lewis’ natural assumption, abstracting from agency problems between managers and shareholders, that managers maximize equity value in this stage of the game, when debt levels are taken as given.
1.3.1 BL Output Equilibrium

Suppose that the discount factor $\beta$ is equal to zero ($\delta = 1$), which means that both firms ignore the second-period output competition and focus on only the first-period output market. This is exactly the same game as in the BL model. Given debt levels $(d_i, d_j)$, each firm chooses the first-period output, $q_{i1}$, to maximize the first period stock present value, $S_{i1}$, which is the whole present value to stockholders, $\hat{V}_i$, at $\beta = 0$. Assuming an interior solution, when the discount factor is zero, the choice of the period output for firm $i$ is obtained by setting the derivative of (1.2.12), with respect to $q_{i1}$, equal to zero.

$$\frac{\partial V_i}{\partial q_{i1}} = \frac{\partial S_{i1}}{\partial q_{i1}} = \int_{z_{i1}}^{1} \left( \frac{\partial \Pi_{i1}(q_{i1}, q_{j1}, \varepsilon_{i1})}{\partial q_{i1}} \right) d\varepsilon_{i1} = 0 \quad (1.3.2)$$

The second-order condition is,

$$\frac{\partial^2 V_i}{\partial^2 q_{i1}} = \frac{\partial^2 S_{i1}}{\partial^2 q_{i1}} < 0 \quad (1.3.3)$$

In addition, I require that

$$\frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} = \frac{\partial^2 S_{i1}}{\partial q_{i1} \partial q_{j1}} < 0 \quad (1.3.4)$$

$$\frac{\partial^2 V_i}{\partial^2 q_{i1}} \frac{\partial^2 V_i}{\partial^2 q_{i1}} - \frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} \frac{\partial^2 V_j}{\partial q_{j1} \partial q_{i1}} = \frac{\partial^2 S_{i1}}{\partial^2 q_{i1}} \frac{\partial^2 S_{i1}}{\partial^2 q_{i1}} - \frac{\partial^2 S_{i1}}{\partial q_{i1} \partial q_{j1}} \frac{\partial^2 S_{j1}}{\partial q_{j1} \partial q_{i1}} > 0 \quad (1.3.5)$$

The Nash output equilibrium of this output market is obtained from the simultaneous solution of (1.3.2) for firms $i$ and $j$ at, say BL output equilibrium, $(q_{i1}^{BL}(d_i, d_j), q_{j1}^{BL}(d_i, d_j))$, given debt levels $(d_i, d_j)$. 
To examine how debt levels affect the BL equilibrium, I totally differentiate the first-order conditions (1.3.2).

\[
\frac{\partial^2 S_{i1}}{\partial q_{i1}} dq_{i1} + \frac{\partial^2 S_{i1}}{\partial q_{i1} \partial d_i} dq_{i1} + \frac{\partial^2 S_{i1}}{\partial q_{i1} \partial d_i} dd_i = 0
\]

(1.3.7)

\[
\frac{\partial^2 S_{j1}}{\partial q_{j1} \partial q_{i1}} dq_{i1} + \frac{\partial^2 S_{j1}}{\partial q_{j1} \partial d_i} dq_{i1} + \frac{\partial^2 S_{j1}}{\partial q_{j1} \partial d_i} dd_i = 0
\]

Since \( \partial S_{j1}/\partial q_{j1} \) does not depend on \( d_i \), \( \partial^2 S_{j1}/\partial q_{j1} \partial d_i = 0 \) at \( \beta = 0 \). Using Cramer’s rule with (1.3.7), we can show comparative static effects, \( dq_{i1}/dd_i \) and \( dq_{j1}/dd_i \), as follows:

\[
\frac{dq_{i1}}{dd_i} = -\frac{1}{H} \left\{ \frac{\partial^2 S_{i1}}{\partial q_{i1} \partial d_i} \frac{\partial^2 S_{j1}}{\partial q_{j1} \partial d_i} \right\}
\]

(1.3.8)

\[
\frac{dq_{j1}}{dd_i} = \frac{1}{H} \left\{ \frac{\partial^2 S_{i1}}{\partial q_{i1} \partial d_i} \frac{\partial^2 S_{j1}}{\partial q_{j1} \partial d_i} \right\}
\]

Since \( H = \frac{\partial^2 S_i}{\partial q_{i1}^2} - \frac{\partial^2 S_i}{\partial q_{i1} \partial q_{j1}} \frac{\partial^2 S_j}{\partial q_{j1}^2} \) is positive from (1.3.5), while \( \frac{\partial^2 S_{i1}}{\partial q_{i1} \partial d_i} < 0 \) from (1.3.2) and \( \frac{\partial^2 S_{j1}}{\partial q_{j1} \partial q_{i1}} > 0 \) from (1.3.3)\(^4\), all that is needed is to sign \( \frac{\partial^2 S_{i1}}{\partial q_{i1} \partial d_i} \).

\[
\frac{\partial^2 S_{i1}}{\partial q_{i1} \partial d_i} = - \left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial q_{i1}} \right) \left( \frac{\partial z_{i1}}{\partial d_i} \right) = - \left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial q_{i1}} \right) \left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial \varepsilon_{i1}} \right)
\]

(1.3.9)

The denominator is positive by (1.3.1.1). Since \( \partial \Pi_{i1}(\varepsilon_{i1})/\partial q_{i1} \) is increasing in \( \varepsilon_{i1} \), \( \partial \Pi_{i1}(z_{i1})/\partial q_{i1} \) must be negative by (1.3.2), implying that the expression in (1.3.9) is positive. Combining this with (1.3.8) yields

\[
\frac{dq_{i1}}{dd_i} > 0, \quad \frac{dq_{j1}}{dd_i} < 0
\]

(1.3.10)

\(^4\)These conditions do hold at least if \( \varepsilon_{it} \) is uniformly distributed, demand is linear, and marginal cost is constant.
The signs in (1.3.10) represent the key insight to be taken from the Brander and Lewis analysis. The position of firm $i$’s reaction function depends on the debt level of firm $i$. A high level of debt $d_i$ is optimal for firm $i$ to produce more in response to any output from its rival, firm $j$. An increase in a firm’s own debt level removes states of low marginal return from the region in which debt holders are residual claimants. In other words, the reaction function is shifted out. In effect, debt financing serves to commit the firm to an aggressive stance in the output market. Therefore, if both firms hold positive debt levels when the discount factor is zero, then, ignoring the second-period market, firms choose aggressive output levels so that the leveraged output equilibrium is higher than the unleveraged Cournot output equilibrium. Hence the stock returns of two leveraged firms will be worse than those of unleveraged firms. This implies that debt has a pro-competitive effect in the one-shot output market.

1.3.2 Output Equilibrium in Repeated Games

Since the manager does not consider gains of the future stage in the BL model, the expected present value of stock returns, $V$, which is the current-period stock returns $S_{i1}$ in their model, is affected only by the current period bankruptcy rate $z_{i1}$. In the repeated production game, however, the expected present value of stock returns is affected not only by its own bankruptcy rate $z_{i1}$ but also by the rivals’ bankruptcy rate, $z_{j1}$ because the likelihood of monopoly position in the next period depends on the rival’s current period bankruptcy
rate. Therefore, in the repeated production game, the manager is connected with the next period market and has an incentive to reduce the rival’s survival rate if such a strategy increases the expected stock return. Hence the predation effect is considered as well as the limited liability effect in the repeated game, while only the limited liability effect is considered in the BL model. Therefore, the first-period output strategy is different from the BL output equilibrium, unless the managers of the two firms do not consider the next period market.

1.3.2.1 Output Equilibrium in the Second Period

If firm $i$ paid its debt obligation at the first period but firm $j$ did not, then firm $i$ would be the monopolist in the second-period output market. Hence firm $i$ chooses the monopoly output level, $q_{i2}^M$, which is constant over $d_i$ and receives the monopoly stock return $S^M_{i2}$ in the second period as shown in (1.2.4).

If both firms met their debt obligation in the first period, then they would compete again in the second period duopoly game. In this subgame, firm $i$ chooses $q_{i2}$, which maximizes (1.2.6) given debt levels $(d_i, d_j)$. Note that the second-period duopoly output equilibrium $(q_{i2}(d_i, d_j), q_{j2}(d_i, d_j))$ is definitely the same as the first period output equilibrium with $\beta = 0$, $(q_{i1}^{BL}(d_i, d_j), q_{j1}^{BL}(d_i, d_j))$, because the first-order condition in (1.2.6) corresponds to (1.3.2). Therefore, the second period duopoly output levels are the same as the BL output equilibrium $(q_{i2}^{BL}(d_i, d_j), q_{j2}^{BL}(d_i, d_j))$ given $(d_i, d_j)$. The second-period duopoly stock return, $S^D_{i2}$ is replaced by $S^{BL}_{i2}(d_i, d_j)$. Therefore, additionally
note that the second-period output levels $q^B_{i2}$ and $q^M_{i2}$ are independent of the 
first-period output level, $q_{i1}$, but depend on debt levels $(d_i, d_j)$.

### 1.3.2.2 Output Equilibrium in the First Period

In the first period the stockholders figure out the second-period output 
equilibria, $(q^B_{i2}, q^B_{j2})$ and $q^M_{i2}$ given debt levels $(d_i, d_j)$; subsequently stockholders 
can figure out the stock returns of firm $i$, $S^M_{i2}$ and $S^B_{i2} = S^D_{i2}(q^B_{i2}, q^B_{j2})$. 
Since $q^B_{i2}$ and $q^M_{i2}$ are independent of $q_{i1}$, firm $i$ faces the following maximization 
objective function in the first period:

$$V_i(q_{i1}, q_{j1}|d_i, d_j) = S_{i1} + \beta(1 - z_{i1}) \left[ (z_{j1})S^M_{i2} + (1 - z_{j1})S^B_{i2} \right] \quad (1.3.11)$$

Assuming an interior solution, the choice of the first-period output for 
firm $i$ is obtained by setting the derivative of (1.3.11), with respect to $q_{i1}$, equal 
to zero:

$$\frac{\partial V_i}{\partial q_{i1}} = \frac{\partial S_{i1}}{\partial q_{i1}} + \beta T_i = 0 \quad (1.3.12)$$

where $T_i$ is denoted by,

$$T_i \equiv (1 - z_{i1}) \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left[ S^M_{i2} - S^B_{i2} \right] - \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left[ (z_{j1})S^M_{i2} + (1 - z_{j1})S^B_{i2} \right] \quad (1.3.13)$$

The first-period output equilibrium is obtained from the simultaneous solution 
of (1.3.12) for $i, j$. The level of $\beta$ does not affect the sign of $T_i$ and affects 
only the size of the gap between the optimal first-period output, $q_{i1}$, and 
the BL output equilibrium, $q^B_{i1}$. The sign of $T_i$ is ambiguous theoretically 
because $\partial z_{j1}/\partial q_{i1}$ and $\partial z_{i1}/\partial q_{i1}$ are positive by (1.3.1.3) and (1.3.1.4). The
first term of $T_i$ represents the marginal benefits of predation, and the second term of $T_i$ represents the marginal costs of predation. Increases in firm $i$’s first period production $q_{i1}$ raise not only the rival $j$’s bankruptcy rate, $z_{j1}$, but also increase its own bankruptcy rate, $z_{i1}$. Hence, if $T_i > 0$, then the first term of (1.3.12), $\partial S_{i1}/\partial q_{i1}$, must be negative so that the first-period output equilibrium $q_{i1}(d_i, d_j)$ is greater than the BL output equilibrium $q_i^{BL}$. If $T_i < 0$, then it is true vise versa. If firm $i$ chooses zero debt and firm $j$ chooses a certain positive level of debt $d_j$, then firm $i$’s first-period bankruptcy rate $z_{i1}$ is zero. Therefore, $[T_i]_{d_i=0}$ is given by,

$$[T_i]_{d_i=0} = \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) [S_{i2}^M - S_{i2}^{BL}] > 0 \quad (1.3.14)$$

Then (1.3.14) implies that the unleveraged firm $i$’s first-period output level, $q_{i1}$, is greater than the BL output level, $q_i^{BL}$, given debt levels $(0, d_j)$. Note that though $[T_i]_{d_i=0}$ is positive, it does not mean that the equilibrium $q_{i1}(0, d_j)$ is greater than the Cournot equilibrium $q^C$, which is the BL output equilibrium of two unleveraged firms. It depends on the discount factor $\beta$. If $\beta$ is positive but very close to zero, then $\beta T_i$ is a small positive term such that $\partial S_{i1}/\partial q_{i1}$ is negative but very close to zero in the first-order condition (1.3.12). It means that although the first-period equilibrium $q_{i1}(0, d_j)$ is slightly greater than $q_i^{BL}(0, d_j)$, it is still smaller than the Cournot equilibrium $q^C$ because $q^C$ is relatively much greater than $q_i^{BL}(0, d_j)$. As $\beta$ rises, however, the manager of firm $i$ has an incentive to increase output in order to raise the rival’s bankruptcy rate, even though doing so reduces the current period returns.
On the other hand, if the rival firm \( j \) chooses zero debt and firm \( i \) chooses a certain positive level of debt \( d_i \), then the firm \( j \)'s first period bankruptcy rate \( z_{j1} \) is zero and \( \partial z_{j1} / \partial q_{i1} \) is also zero. Hence, \([T_i]_{d_j=0} \) is given by,

\[
[T_i]_{d_j=0} = - \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) S_{i2}^{BL} < 0
\]  

(1.3.15)

This implies that the first-period output level of leveraged firm \( i \), \( q_{i1} \), is less than the BL output level \( q_i^{BL} \), given debt levels \((d_i, 0)\). As \( \beta \) rises, the gap \( q_i^{BL} - q_{i1} \) becomes bigger. Hence, with a higher \( \beta \) level, the leveraged firm produces less in the first-period output market in order to avoid predation by the unleveraged rival.

The second-order condition of (1.3.11) is

\[
\frac{\partial^2 V_i}{\partial^2 q_{i1}} < 0
\]  

(1.3.16)

In addition I require that

\[
\frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} < 0
\]  

(1.3.17)

\[
\frac{\partial^2 V_i}{\partial^2 q_{i1}} \cdot \frac{\partial^2 V_j}{\partial^2 q_{j1}} - \frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} \cdot \frac{\partial^2 V_j}{\partial q_{j1} \partial q_{i1}} > 0
\]  

(1.3.18)

In the BL model, the leveraged firm chooses an aggressive output strategy because of the limited liability effect, and the unleveraged firm chooses a

---

\(^5\)It is well known that even in the simplest models, conditions analogous to (1.3.17) and (1.3.18) can be violated by feasible demand and cost structures, and that is certainly true here. One case in which these conditions do hold, however, is if \( \varepsilon_{it} \) is uniformly distributed, demand is linear, and marginal cost is constant.
passive output level which maximizes the expected profit of one-shot output market. In the repeated game, the strategic bankruptcy effect is involved, and the importance of this effect emerges as $\beta$ rises while the limited liability effect decreases. At a high $\beta$ level, despite sacrificing the current-period profits, the manager of an unleveraged firm chooses an aggressive output strategy to receive a high likelihood of a monopoly position in the next period. On the other hand, the manager of the leveraged firm chooses a passive output level in order to reduce the bankruptcy rate and receive the additional next-period stock returns. Although the limited liability effect still urges the manager to choose an aggressive strategy at a high $\beta$ level, the effect is not as strong as the strategic bankruptcy effect.

1.3.2.3 Comparative Statics

To examine how debt levels affect best responses of firms, we need to totally differentiate first-order conditions (1.3.12) for $i$ and $j$.

\[
\frac{\partial^2 V_i}{\partial q_{i_1}} dq_{i_1} + \frac{\partial^2 V_i}{\partial q_{i_1} \partial q_{j_1}} dq_{j_1} + \frac{\partial^2 V_i}{\partial q_{i_1} \partial d_i} dd_i = 0 \quad (1.3.19)
\]

\[
\frac{\partial^2 V_j}{\partial q_{j_1} \partial q_{i_1}} dq_{i_1} + \frac{\partial^2 V_j}{\partial q_{j_1} \partial d_i} dq_{j_1} + \frac{\partial^2 V_j}{\partial q_{j_1} \partial d_i} dd_i = 0 \quad (1.3.20)
\]

The first point to make is that $\partial^2 V_j/\partial q_{j_1} \partial d_i$ is non-zero at $\beta > 0$, while it is zero at $\beta = 0$. Putting (1.3.19) and (1.3.20) in matrix form and using Cramer’s
rule to solve for comparative static effects, \( dq_{i1}/dd_i \) and \( dq_{j1}/dd_i \) yields

\[
\frac{dq_{i1}}{dd_i} = -\left( \frac{1}{G} \right) \left\{ \frac{\partial^2 V_i}{\partial q_{i1} \partial d_i} \frac{\partial^2 V_i}{\partial d_i} - \frac{\partial^2 V_j}{\partial q_{j1} \partial d_i} \frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} \right\} \\
\frac{dq_{j1}}{dd_i} = -\left( \frac{1}{G} \right) \left\{ \frac{\partial^2 V_i}{\partial q_{i1} \partial d_i} \frac{\partial^2 V_i}{\partial q_{j1} \partial d_i} - \frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} \right\}
\]

(1.3.21)  
(1.3.22)

where \( G = \frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} - \frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} \frac{\partial^2 V_i}{\partial q_{j1} \partial d_i} > 0 \) from (1.3.18). Signs of \( \partial^2 V_i/\partial q_i \partial d_i \) and \( \partial^2 V_j/\partial q_j \partial d_i \) are theoretically ambiguous, because they depend on \( \beta \), demand function, and cost function. Therefore, the debt’s effects on the first-period output equilibrium are ambiguous. The situation is different in the BL model where debt obviously increases the firm’s own output level and decreases the rival’s output level. Although the signs of debt’s effects on output equilibrium is not unilaterally obvious, I can still figure out the comparative statics of the unleveraged firm. At \( d_i = 0 \), derivatives \( \frac{\partial^2 V_i}{\partial q_{i1}} \), \( \frac{\partial^2 V_i}{\partial q_{j1}} \), \( \frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} \) and \( \frac{\partial^2 V_i}{\partial q_{j1} \partial q_{i1}} \) are independent of \( \beta \) as follows\(^6\).

\[
\left[ \frac{\partial^2 V_i}{\partial q_{i1}^2} \right]_{d_i=0} = \int_0^1 \frac{\partial^2 \Pi_{i1}}{\partial q_{i1}^2} d\varepsilon_{i1} < 0
\]

(1.3.23.1)

\[
\left[ \frac{\partial^2 V_j}{\partial q_{j1}^2} \right]_{d_i=0} = -\left( \frac{\partial \Pi_{j1}(z_{j1})}{\partial q_{j1}} \right) \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) + \int_{z_{j1}}^1 \frac{\partial^2 \Pi_{j1}}{\partial q_{j1}^2} d\varepsilon_{j1} < 0
\]

(1.3.23.2)

\[
\left[ \frac{\partial^2 V_i}{\partial q_{j1} \partial q_{i1}} \right]_{d_i=0} = \int_0^1 \frac{\partial^2 \Pi_{i1}}{\partial q_{i1} \partial q_{j1}} d\varepsilon_{i1} < 0
\]

(1.3.23.3)

\[
\left[ \frac{\partial^2 V_j}{\partial q_{i1} \partial q_{j1}} \right]_{d_i=0} = -\left( \frac{\partial \Pi_{j1}(z_{j1})}{\partial q_{j1}} \right) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) + \int_{z_{j1}}^1 \frac{\partial^2 \Pi_{j1}}{\partial q_{i1} \partial q_{j1}} d\varepsilon_{j1} < 0
\]

(1.3.23.4)

\(^6\)See Appendix A.1.
However, derivatives $\frac{\partial^2 V_i}{\partial q_{i1} \partial d_i}$ and $\frac{\partial^2 V_i}{\partial q_{j1} \partial d_i}$ are decreasing on the $\beta$ level, as follows.

$$
\left[ \frac{\partial^2 V_i}{\partial q_{i1} \partial d_i} \right]_{d_i=0} = -\left( \frac{\partial \Pi_{i1}}{\partial q_{i1}} \right) \left( \frac{dz_{i1}}{d_i} \right) - \beta \left( \frac{dz_{j1}}{d q_{j1}} \right) \left( \frac{dz_{i1}}{d d_i} \right) \left[ S^M_{i2} - S^{BL}_{i2} \right]
$$

$$
\left[ \frac{\partial^2 V_i}{\partial q_{j1} \partial d_i} \right]_{d_i=0} = \beta(1 - z_{j1}) \left( \frac{d^2 z_{i1}}{d q_{j1} dd_i} \right) \left( \frac{dz_{i1}}{d d_i} \right) \left[ S^M_{i2} - S^{BL}_{i2} \right]
$$

The first term of (1.3.25) represents the limited liability effect in the first period and the sign of this term is positive, which means that the bankruptcy point $z_{i1}$ rises and the unleveraged firm $i$ chooses a more aggressive output level as the unleveraged firm $i$ starts to issue debt. The second term of (1.3.25) involves the strategic bankruptcy effect, which implies that issuing debt raises the costs of firm $i$’s quantity predation on the rival $j$.

Once the unleveraged firm $i$ issues debt, its bankruptcy rate is also raised by its output increases. By increasing its output $q_{i1}$ to prey upon its rival $j$, its own bankruptcy rate $z_{i1}$ is also raised so that the expected returns from the second-period market simultaneously fall. If an unleveraged firm $i$ does not issue any debt, then when firm $i$ increases output level $q_{i1}$ to reduce the rival firm’s survival rate $1 - z_{j1}$, firm $i$ loses the current-period profit $\Pi_{i1}$

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but does not lose its survival rate \(z_{i1} = 0\). The third term of (1.3.25), which has a negative sign, represents the limited liability effect in the second period duopoly game. Hence the first term of (1.3.25) is positive, but the two other terms are negative.

The first term of (1.3.26) is positive and represents the strategic bankruptcy effect, which means that issuing debt \(d_i\) makes the firm \(i\) vulnerable to rival \(j\)’s predation. If the unleveraged firm \(i\) starts to issue a positive debt level, then its bankruptcy point, \(z_{i1}\), rises from zero. The rival \(j\) then chooses a more aggressive output level in order to prey upon firm \(i\) and also reduce firm \(i\)’s survival rate, \(1 - z_{i1}\). The second term of (1.3.26) represents the costs of predation and implies that the rival \(j\) does not have to increase output much for the purpose of predation if firm \(i\) raises its debt level, because increases in debt make the firm more vulnerable. The third term of (1.3.26), which is positive, represents the effect of the rival’s debt on the rival’s second-period output level.

The derivative (1.3.25), \[\left[ \frac{\partial^2 V_i}{\partial q_{i1} \partial d_i} \right]_{d_i=0}\], is positive at \(\beta = 0\), and rapidly decreases as \(\beta\) rises because the positive first term is independent of \(\beta\) while the remaining two negative terms are correlated with \(\beta\). The other derivative (1.3.26), \[\left[ \frac{\partial^2 V_j}{\partial q_{j1} \partial d_i} \right]_{d_i=0}\], is zero at \(\beta = 0\), and the sign of (1.3.26) is ambiguous at any positive \(\beta\) because it depends on the sum of three terms of (1.3.26). Since the sizes of derivatives (1.3.23.1) to (1.3.23.4) are very small relative to the derivatives (1.3.25) and (1.3.26), the comparative static of the effect of debt on output depends on magnitudes of derivatives (1.3.25) and (1.3.26).
If the derivative (1.3.26) is positive, the comparative static $\left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0}$ falls as $\beta$ rises and as the comparative static $\left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0}$ rises. Even if (1.3.26) is negative, since the second term of (1.3.25) is less than the second term of (1.3.26)\(^7\), the comparative static (1.3.21) falls as $\beta$ rises and as the comparative static (1.3.22) $\left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0}$ rises. Hence $\left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0}$ is decreasing and $\left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0}$ is increasing in the discount factor $\beta$;

$$\frac{\partial \left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0}}{\partial \beta} < 0, \quad \frac{\partial \left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0}}{\partial \beta} > 0 \quad (1.3.27)$$

Therefore, as $\beta$ approaches infinity, comparative statics of an unleveraged firm are given by

$$\lim_{\beta \to \infty} \left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0} < 0 \quad (1.3.28.1)$$

$$\lim_{\beta \to \infty} \left[ \frac{dq_{ij}}{dd_i} \right]_{d_i=0} > 0 \quad (1.3.28.2)$$

These comparative statics (1.3.28.1) and (1.3.28.2) result from only the strategic bankruptcy effect. The limited liability effect is represented as the first term of (1.3.25) vanishes as $\beta$ approaches infinity\(^8\). Terms representing the strategic bankruptcy effect remain. As $\beta$ approaches infinity, if an unleveraged firm $i$ issues a tiny amount of debt; the costs of predation and production

---

\(^7\)Because increases in output level raise the rival’s bankruptcy rate much more rather than its own bankruptcy rate; that is $dz_{ij}/dq_{ij} > dz_{ij}/dq_{ij}$.

\(^8\)The strategic bankruptcy effect dominates the limited liability effect as $\beta$ goes to infinite.
increase tremendously because increases in output level $q_{i1}$ also fatally reduce its survival rate $1 - z_{i1}$. Additionally, if an unleveraged firm issues a tiny amount of debt, the rival $j$ increases output level $q_{j1}$ to prey upon the firm $i$ as $\beta$ approaches infinity. Therefore, if an unleveraged firm $i$ starts to issue debt $d_i$ at a high $\beta$ level, the debt reduces its first-period output level $q_{i1}$ and induces firm $j$ to raise its first-period output level $q_{j1}$ as shown in (1.3.28.1) and (1.3.28.2).

### 1.4 Selection of Debt Levels

The previous sections have explained the dependance of industry output levels on debt structure, treating debt as a predetermined or exogenous variable. In this section we describe the determinants of the debt structure. We assume that the manager of each firm is free to choose whatever output level he desires after debt is issued. In particular, debt covenants, which would restrict the manager’s strategy decisions, are not considered. Debt covenants and other precommitment devices might be, of course, used for strategic purposes, but here we focus exclusively on the strategic effects through financial structure.

For the selection of debt levels, I will adopt the Nash equilibrium in debt levels, subject to the constraint that firms and bondholders correctly anticipate the resolution of the Nash equilibrium in the repeated output game.
1.4.1 The Whole Firm Value Maximization

A firm needs $I$ in investments, and the corporate managers select a financing provision to finance $I$. The production levels cannot be observed by bondholders at the time the bonds are issued, but bondholders are rational and so understand the nature of the incentives management faces as well as the nature of the competition in the repeated production game. Hence, for any pair of promised payments $(d_i, d_j)$ and any discount factor $\beta$, the creditors can anticipate the production levels accurately. Suppose that the corporate managers determine the corporate capital structure. This may be done by supposing that the managers can issue any combination of new equity and new debt. For simplicity’s sake, I assume that both firms are initially unleveraged. The financing condition facing each manager is of the form that the sum of the new equity issue, $V_i^{new}$ and the new debt issue, $W_i$ equals the investment, $I$ (i.e. $V_i^{new} + W_i = I$). By backward induction, I deduce that given any debt level, the creditors correctly anticipate output levels ($q_{i1}$, $q_{j1}$, $q_{i2}$, $q_{j2}$, $q_{i2}^M$, $q_{j2}^M$) and value the debt issue appropriately. When the manager makes the capital structure decision, the problem becomes one in which the bond issues are made given that the bondholders know the consequences of the capital structure choice. The same thing must be said about any new equity holders.

I will use $\alpha_i$ to denote the size of new equity issued divided by the total equity issued. Then $\alpha_i = n_i / (N_i + n_i)$, where $n_i$ is the number of new equity issued and $N_i$ is the number of old equities issued. Since the firms are assumed to be initially unleveraged, $W_i$ will be used subsequently for the present value
of the new debt issue. Having specified the financing condition, it should follow that any manager with interests aligned to those of the shareholders selects the capital structure of the firm to

$$\begin{align*}
\text{Max} & \quad V_i^{\text{old}} \quad \text{s.t.} \quad V_i^{\text{new}} + W_i = I \\
\end{align*}$$

(1.4.1)

where $V_i^{\text{old}}$ represents the old shareholders’ stake in the corporation and $V_i^{\text{New}}$ represents the new shareholders’ stake in the corporation.

$$\begin{align*}
V_i^{\text{old}} &= \frac{N_i}{N_i + n_i} V_i = (1 - \alpha_i) V_i \\
V_i^{\text{new}} &= \frac{n_i}{N_i + n_i} V_i = \alpha_i V_i
\end{align*}$$

Now consider the manager’s financial structure decision. The constrained maximization problem in (1.4.1) is equivalently expressed in Lagrange form as

$$\begin{align*}
\text{Max} & \quad L(\alpha_i, \mu) \equiv (1 - \alpha_i) V_i + \mu (\alpha_i V_i + W_i - I) \\
\end{align*}$$

(1.4.2)

The condition for a maximum is as follows:

$$\frac{\partial L}{\partial \alpha_i} = -V_i + \mu V_i = 0 \iff \mu = 1$$

Therefore, the financial decision problem is changed to the following maximization objective:

$$\begin{align*}
\max_{\{d_i\}} & \quad V_i + W_i - I
\end{align*}$$

(1.4.3)

The maximization problem (1.4.3) explains that the manager selects the debt level which maximizes the firm’s total present value.
1.4.2 Optimal Financial Decision in the BL Model

In the Brander and Lewis model, there is only a one period output market; that means $\beta = 0$, and both firms’ managers have an incentive to maximize one-period profits, $S_{i1} + D_{i1}$:

$$\text{Max } V_i + W_i - I = S_{i1} + D_{i1} - I$$

(1.4.4)

$$\{d_i\}$$

This means that the manager chooses the debt level which maximizes one-shot profits without concern for the next period’s game. The first-order condition (1.4.4) is given by

$$\frac{d(S_{i1} + D_{i1})}{dd_i} = \left(\int_0^{z_{i1}} \frac{\partial \Pi_{i1}}{\partial q_{i1}} d\varepsilon_{i1}\right) \left[\frac{dq_{i1}}{dd_i}\right] + \left(\int_0^{1} \frac{\partial \Pi_{i1}}{\partial q_{j1}} d\varepsilon_{j1}\right) \left[\frac{dq_{j1}}{dd_i}\right] = 0$$

(1.4.5)

The first term on the right-hand side (RHS) of (1.4.5) is negative because of assumption (1.2.3) and the first-order condition (1.3.2) and because $dq_{i1}/dd_i > 0$ in (1.3.10). The second term on the RHS of (1.4.5), however, is positive because $\partial \Pi_{i1}/\partial q_j$ is negative for all states of the world and $dq_{j1}/dd_i$ is also negative in (1.3.10). The first negative term is the marginal cost of leverage. Increased leverage increases the manager’s incentive to shift risk to the bondholders by increasing output. The second positive term is the marginal benefit, due to the competitor’s reduced output, which means the gains due to the competitive advantage associated with leverage.

If the firm holds zero debt, then $z_{i1} = 0$, and the first term on the RHS
of (1.4.5) disappears, leaving the positive derivative

$$\left[ \frac{d(S_{i1} + D_{i1})}{dd_i} \right]_{d_i=0} = \left( \int_0^1 \frac{\partial \Pi_{i1}}{\partial q_{i1}} d\xi_2 \right) \left[ \frac{d q_{i1}}{dd_i} \right] > 0 \quad (1.4.6)$$

Expression (1.4.6) means that debt is the dominant strategy; as a result, the firms hold positive debt levels. Therefore, symmetric positive debt levels ($d^{BL}, d^{BL}$) are selected and satisfy (1.4.5). Hence the stock returns of two leveraged firms will be worse than those of unleveraged firms. This implies that debt has a pro-competitive effect in the one-shot output market.

1.4.3 Optimal Financial Decision in Repeated Games

The whole firm’s present value is denoted by $Y_i$, ($Y_i \equiv V_i + W_i$). Then the first order condition of problem (1.4.3) is then

$$\frac{dY_i}{dd_i} = \frac{dV_i}{dd_i} + \frac{dW_i}{dd_i} = 0 \quad (1.4.7)$$

The combined present value is the expected present value of two period operating profits over all states of the world. Issuing debt is strictly a break-even transaction for the firm, except for the fact that the firm cannot reach the next period production if it goes bankrupt in the first period.

The first-order condition (1.4.7) is given by\(^9\)

\(^9\)See Appendix A.2.
\[
\frac{dY_i}{dd_i} = \left[ \frac{dq_{i1}}{dd_i} \right] \left[ \int_0^{z_{i1}} \frac{\partial \Pi_{i1}}{\partial q_{i1}} d\varepsilon_{i1} \right]
- \beta \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left\{ (z_{i1})d_i + (1 - z_{i1}) \int_0^{\varepsilon_{i2}} \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} + (1 - z_{i1})(1 - z_{i2})d_i \right\}
- \beta(1 - z_{i1}) \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left\{ (z_{i2})d_i - \int_0^{\varepsilon_{i2}} \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right\}
+ \left[ \frac{dq_{j1}}{dd_i} \right] \left[ \int_0^1 \frac{\partial \Pi_{i1}}{\partial q_{j1}} d\varepsilon_{i1} \right]
- \beta \left( \frac{\partial z_{i1}}{\partial q_{j1}} \right) \left\{ (z_{j1}) \int_0^1 \Pi_{i2}^M(\varepsilon_{i2}) d\varepsilon_{i2} + (1 - z_{j1}) \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right\}
- \beta(1 - z_{i1}) \left( \frac{\partial z_{i1}}{\partial d_i} \right) \left\{ \int_0^1 \Pi_{i2}^M(\varepsilon_{i2}) d\varepsilon_{i2} + (1 - z_{j1}) \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right\}
- \beta(1 - z_{i1})(1 - z_{i1}) \left[ \left( \int_0^{\varepsilon_{i2}} \frac{\partial \Pi_{i2}}{\partial q_{i2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{i2}}{dd_i} \right] + \left( \int_0^1 \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{j2}}{dd_i} \right] \right]
= 0
\]

Note that the last term is zero at the BL debt levels which maximize the second period profits of the duopoly subgame because of the first order condition (1.3.29).

To examine how the optimal debt levels are affected in the repeated game, we have to compare the marginal effect of an increase in \( d_i \) on the firm \( i \) when it does hold debt.
\[
\left[ \frac{dY_i}{dd_i} \right]_{d_i=0} = A \left[ \frac{dq_{j1}}{dd_i} \right]_{d_i=0} - \beta B + \beta C \tag{1.4.9}
\]

where

\[
A \equiv \int_0^1 \frac{\partial \Pi_{i1}}{\partial q_{j1}} d\varepsilon_{i1} - \beta \left\{ \int_0^1 \Pi_{i2}^M(\varepsilon_{i2}) - \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right\} \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right)_{d_i=0} < 0
\]

\[
B \equiv \left( z_{j1} \int_0^1 \Pi_{i2}^M(\varepsilon_{i2}) d\varepsilon_{i2} + (1 - z_{j1}) \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right) \left( \frac{\partial z_{i1}}{\partial d_i} \right)_{d_i=0}
\]

\[
C \equiv (1 - z_{j1}) \left( \int_0^1 \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{j2}}{dd_i} \right]_{d_i=0}
\]

The sign of \( \left[ \frac{dq_{j1}}{dd_i} \right]_{d_i=0} \) determines the debt-financing decision\(^{10}\); if \( \left[ \frac{dY_i}{dd_i} \right]_{d_i=0} > 0 \), firm \( i \) holds debt to finance, while it issues no debt if \( \left[ \frac{dY_i}{dd_i} \right]_{d_i=0} \leq 0 \). Since the sign of \( \left[ \frac{dq_{j1}}{dd_i} \right]_{d_i=0} \) is ambiguous for a certain \( \beta \) level, the sign of \( \left[ \frac{dY_i}{dd_i} \right]_{d_i=0} \) is not obvious because \( \left[ \frac{dq_{j1}}{dd_i} \right]_{d_i=0} \) depends on \( \beta \) level as shown in (1.3.27), (1.3.28.1) and (1.3.28.2). The term \( A \) is negative and decreasing on \( \beta \) (\( \partial A/\partial \beta < 0 \)), and the terms \( B \) and \( C \) are positive.

Since \( \left[ \frac{dq_{j1}}{dd_i} \right]_{d_i=0} \) is negative at \( \beta = 0 \), \( \left[ \frac{dY_i}{dd_i} \right]_{d_i=0, \beta=0} > 0 \). Then firm \( i \) wants to hold a positive debt level when \( \beta = 0 \). In addition, the differentiation of (1.4.9) with respect to \( \beta \) is positive at \( \beta = 0 \) as follows:

\[
\left[ \frac{\partial}{\partial \beta} \left[ \frac{dY_i}{dd_i} \right]_{d_i=0} \right]_{\beta=0} = \left( \frac{\partial A}{\partial \beta} \right) \left[ \frac{dq_{j1}}{dd_i} \right]_{d_i=0, \beta=0} + A \left[ \frac{\partial}{\partial \beta} \left[ \frac{dq_{j1}}{dd_i} \right]_{d_i=0} \right]_{\beta=0} > 0 \tag{1.4.10}
\]

This equation (1.4.10) shows the strategic bankruptcy effect on the debt equilibrium of the BL model. In the BL model, only the limited liability

\(^{10}\)If \( [dY_i/dd_i]_{d_i=0} > 0 \) for any \( \beta \), then holding debt is the dominant strategy for all \( \beta \), and if \( [dY_i/dd_i]_{d_i=0} \leq 0 \) for any \( \beta \) then unleveraging is the best strategy for any discount factor level.
effect is considered and the strategic bankruptcy effect is ignored. Therefore, equation (1.4.10) shows that, considering the strategic bankruptcy effect, the manager must choose a higher debt level as an optimal debt level in the BL model. Hence it implies that the strategic bankruptcy effect reduces the optimal debt level of the BL model.

As $\beta$ approaches infinity, $\left[ \frac{dq_{01}}{dd_i} \right]_{d_i=0}$ rapidly increases as shown in (1.3.27), and its limit value is negative as shown in (1.3.28.2). The differentiation of (1.4.9) with respect to $\beta$ is negative as $\beta$ approaches to infinity as follows:

$$
\lim_{\beta \to \infty} \frac{\partial}{\partial \beta} \left[ \frac{dy_i}{dd_i} \right]_{d_i=0} = \left( \frac{\partial A}{\partial \beta} \right) \left( \lim_{\beta \to \infty} \left[ \frac{dq_{01}}{dd_i} \right]_{d_i=0} \right) + A \left( \frac{\partial}{\partial \beta} \left[ \frac{dq_{01}}{dd_i} \right]_{d_i=0} \right) - (B - C)
$$

< 0

(1.4.11)

This is the reverse-expression of (1.4.10) and implies that the strategic bankruptcy effect reduces the optimal debt level as $\beta$ approaches to infinity.

Therefore, we can get

$$
\lim_{\beta \to \infty} \left[ \frac{dy_i}{dd_i} \right]_{d_i=0} < 0
$$

(1.4.12)

The expression (1.4.12) shows that no firm wants to issue debt as $\beta$ approaches infinity because the strategic bankruptcy effect emerges so strongly. Hence zero-debt financing, that is, all-equity financing is the optimal financial decision when the strategic bankruptcy effect is huge.

The basic point of the expression (1.4.12) is straightforward. Since the strategic bankruptcy effect is very strong at a high $\beta$ level. If a firm increases
its debt level, then it faces high costs of predation and has to reduce its output level to keep up its survival rate. Additionally, the firm which increases its debt level becomes more vulnerable, and the rival raises its first-period output level to prey upon this firm. Hence if a firm issues any debt, it should lose not only the current-period stock returns but also the next-period expected returns. Therefore, when the strategic bankruptcy effect dominates the limited liability effect, debt loses its limited liability merit, and the optimal debt level falls.

1.5 Simulation

The preceding sections have shown the dependence of output levels in repeated games on debt structure and the optimal conditions for debt selection. They presented a model in which the present value of firm $i$’s equity is represented by $V_i((q_{i1}, q_{j1}), (q_{i2}, q_{j2}), q_{i2}^M | (d_i, d_j))$. The solutions to the first-order conditions (1.3.2) and (1.3.12) are the Nash equilibrium quantities, $(q_{i1}^*, (d_i, d_j), q_{j1}^*(d_i, d_j))$ and $(q_{i2}^*(d_i, d_j), q_{j2}^*(d_i, d_j))$, which are functions of given debt levels, $(d_i, d_j)$. The present value of firm $i$’s equity, then, is $V_i^*(d_i, d_j)$, and the present value of bonds is $W_i^*(d_i, d_j)$. From firm $i$’s perspective, the optimal debt level is that which maximizes the present value of total profits, $Y_i^*(d_i, d_j) = V_i^*(d_i, d_j) + W_i^*(d_i, d_j)$ subject to $d_i \geq 0$. However, optimal debt equilibrium and output equilibrium were ambiguous because the first order conditions depend on the $\beta$, the distribution of random shocks, the demand function, and the cost structure.
In this section, I examine the debt equilibrium and its effects on two period output levels through simulation under restricted conditions: the linear demand function with additive uncertainty and zero marginal cost\textsuperscript{11}:

\[ P_{it}(\varepsilon_{it}, q_{it}, q_{-it}) = a + \varepsilon_{it} - b(q_{it} + q_{-it}), \quad a > 0, \quad b > 0 \]  
(1.5.1)

The profit function is given by,

\[ \Pi_{it}(\varepsilon_{it}, q_{it}, q_{-it}) = [a + \varepsilon_{it} - b(q_{it} + q_{-it})]q_{it} \]  
(1.5.2)

The random shock is uniformly distributed over $[0, 1]$ as assumed in the previous section. The stock return of firm $i$ at a duopoly period $t$, $S_{it}$, is represented by,

\[ S_{it}(q_{it}, q_{jt}, d_i) = \int_{\overline{z}_{it}}^{1} \{ [a + \varepsilon_{it} - b(q_{it} + q_{jt})]q_{it} - d_i \} d\varepsilon_{it} \]

where $z_{it}$ is such that $[a + z_{it} - b(q_{it} + q_{-it})]q_{it} = d_i$, assuming $0 < z_{it} < 1$.

The first-order condition (1.3.2) of the optimal output is changed to

\[ \frac{\partial S_{i1}}{\partial q_{i1}} = \int_{\overline{z}_{i1}}^{1} \left( \frac{\partial \Pi_{i1}(q_{i1}, q_{j1}, \varepsilon_{i1})}{\partial q_{i1}} \right) d\varepsilon_{i1} = 0 \]

\[ \iff a + \frac{1 + z_{i1}}{2} - 2bq_{i1} - bq_{j1} = 0 \]

\[ q_{i1}(q_{j1} | d_i) = \frac{a + 1 - bq_{j1} + \sqrt{(a + 1 - bq_{j1})^2 + 12bd_i}}{6b} \]

The monopoly quantity and profits are, of course, given by,

\[ q_{i2}^{M} = \frac{(a + \frac{1}{2})}{2b} \]

\textsuperscript{11}Since a linear demand function with additive uncertainty and constant marginal cost is one case in which the Brander and Lewis model does hold, I use this case to simulate the model.
\[ \Pi_{i2}^M = \frac{(a + \frac{1}{2})^2}{4b} \]

The first-order condition (1.3.2) of the optimal BL debt level is changed to

\[
\frac{d(S_{i1} + D_{i1})}{dd_i} = \left( \int_0^{z_{i1}} a + \varepsilon_{i1} - 2bq_{i1} - bq_{j1}d\varepsilon_{i1} \right) \left[ \frac{dq_{i1}}{dd_i} \right] - bq_{i1} \left[ \frac{dq_{j1}}{dd_i} \right] = 0
\]

where

\[
\frac{dq_{i1}}{dd_i} = - \left( \frac{1}{H} \right) \left[ (a + z_{j1} - 2bq_{j1} - bq_{i1})(\frac{1}{q_{j1}}) \right] \times \left[ (a + z_{j1} - 2bq_{j1} - bq_{i1})(\frac{d_j}{q_{j1}^2} + b) + 2b(1 - z_{j1}) \right]
\]

\[
\frac{dq_{j1}}{dd_i} = \left( \frac{1}{H} \right) \left[ (a + z_{j1} - 2bq_{j1} - bq_{i1})(\frac{1}{q_{j1}}) \right] \times [b(a + z_{j1} - 2bq_{j1} - bq_{i1}) + b(1 - z_{j1})]
\]

\[
H = \left[ (a + z_{i1} - 2bq_{i1} - bq_{j1})(- \frac{d_i}{q_{i1}^2} + b) + 2b(1 - z_{i1}) \right] \times \left[ (a + z_{j1} - 2bq_{j1} - bq_{i1})(- \frac{d_j}{q_{j1}^2} + b) + 2b(1 - z_{j1}) \right]
\]

For example, letting \( a = 3, \ b = 1.5 \) gives Cournot equilibrium is, then, \( q^C = .7778 \). The BL debt equilibrium, \( d^{BL} = .9421 \), and the BL output equilibrium, \( q^{BL} = .8511 \). The bankruptcy rate at BL equilibrium, \( z^{BL} = .6603 \).
Table 1.1: Simulation Results

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$d^*$</th>
<th>$q_{t=1}^*$</th>
<th>$z_{t=1}^*$</th>
<th>$q_{t=2}^*$</th>
<th>$z_{t=2}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.091</td>
<td>.5192</td>
<td>.7921</td>
<td>.0319</td>
<td>.7779</td>
<td>.0011</td>
</tr>
<tr>
<td>.090</td>
<td>.5215</td>
<td>.7925</td>
<td>.0355</td>
<td>.7783</td>
<td>.0050</td>
</tr>
<tr>
<td>.085</td>
<td>.5333</td>
<td>.7941</td>
<td>.0540</td>
<td>.7805</td>
<td>.0249</td>
</tr>
<tr>
<td>.080</td>
<td>.5458</td>
<td>.7960</td>
<td>.0736</td>
<td>.7829</td>
<td>.0458</td>
</tr>
<tr>
<td>.070</td>
<td>.5740</td>
<td>.8006</td>
<td>.1201</td>
<td>.7881</td>
<td>.0926</td>
</tr>
</tbody>
</table>

,where $d^*(\beta)$ s.t. $\left[ \frac{dY_i}{dd_i} \right]_{d_i=d_j=d^*} = 0$ given $\beta$

(1) $q_{t=2}^*$ is the BL output equilibrium level

When both firms do not hold debt, the simulation result shows,

$$\left[ \frac{dY_i}{dd_i} \right]_{d_i=d_j=0} \geq 0 \text{ for all } \beta \leq .3648$$

At a high $\beta$ level, firms want to stay unleveraged. Firms want to issue debt with low discount factors because of the predation effect. If both firms choose the BL equilibrium debt level, $d^{BL} = .9421$, the simulation shows that the first-order condition (1.4.8) does not hold for any discount factor except $\beta = 0$:

$$\left[ \frac{dY_i}{dd_i} \right]_{d_i=d_j=d^{BL}=.9421} < 0 \text{ for all } \beta \in (0,1]$$

It means that BL debt levels are greater than the symmetric optimal levels of this model. As $\beta$ approaches zero, the symmetric optimal levels which satisfy the first-order condition (1.4.8) increase as shown in Table 1.1.

When $\beta$ is very close to zero (for example $\beta=.0001$), the optimal debt level, $d^*$, and the optimal first period output level, $q_{t1}^*$, are close to BL equilib-
rium levels, $d^{BL} = .9421$ and $q^{BL} = .8511$. It means that as $\beta$ approaches zero, firms hold debt as commitment and raise the output level aggressively. They reduce their debt burden to avoid predation by the rival as $\beta$ approaches 1.

As shown in Table 1.1, the first-period output equilibrium is a little greater than the second-period output levels. This gap between $q^{*}_{t=1}$ and $q^{*}_{t=2}$ represents not only the limited liability effect but also the predation incentive.

However, the simulation result does not support any asymmetric debt equilibrium. Any asymmetric debt combination $(0, d_{j})$ cannot satisfy the first order condition (1.4.8) simultaneously.

These simulation results are consistent with the other examples, in which combinations of $a$ and $b$ which support $0 \leq z_{it} \leq 1$.

1.6 Conclusions

In this paper, I have presented an analysis of the two effects of corporate financing decisions on output strategy in repeated games. I have demonstrated how financial decisions affect output strategies, and by backward induction, how output decisions affect financial decisions through the limited liability effect and the strategic bankruptcy effect. This paper makes the basic point that the output strategy is affected not only by financial structure but also by gains from future stages. The limited liability effect implies that changes in financial structure alter the distribution of returns between debt holders and equity holders and therefore change the output strategy favored by equity.
holders. The strategic bankruptcy effect implies that the possibility of financial distress for each firm is contingent on its financial structure so that the more the firm is financially constrained, the more it will become vulnerable to predation by its rival. In repeated games, shareholders face the trade-off between current-period returns and future gains. The aggressive (passive) output strategy raises (reduces) the current-period returns but reduces (raises) its survival rate.

The central argument of this paper is that the strategic bankruptcy effect in addition to the limited liability effect must be considered in the first-period output decision. I focus on how the strategic bankruptcy effect induces a leveraged firm to lower its output level. When the strategic bankruptcy effect dominates the limited liability effect, the leveraged firm’s equity holders choose the passive output strategy to reduce the bankruptcy in bad states, while the unleveraged rival raises its output level for predation in the first-period. In this case the net effect of debt financing on the output strategy is negative, which gives a theoretical explanation for recent particular empirical evidence. I find that this occurs when the present value of gains from future stage is very high. On the other hand, if the limited liability is strong enough to dominate the strategic bankruptcy effect, the debt financing has a positive effect on the output decision. Therefore the firms choose a positive debt level strategically in this case. Additionally, I have pointed out that the optimal debt level is lower than it is in the Brander and Lewis model because the strategic bankruptcy effect emerges in repeated production games.
Chapter 2

Repeated Oligopoly Games and Financial Structure with Perfectly Correlated Shocks

2.1 Introduction

As Brander and Lewis (1986) explain, there are two main ways in which the financial structure can affect output markets. The first one is the limited liability effect of debt financing. Leveraged firms have an incentive to pursue output strategies that raise returns in good states and lower returns in bad states. The basic point is that shareholders ignore reductions in returns in bankruptcy states, since bondholders become the residual claimants. As debt levels change, the distribution of returns to shareholders over the different states changes, which in turn changes the output strategy favored by shareholders. Therefore, a high leveraged firm concentrating on good states selects a more aggressive production and it commits an unleveraged rival to a passive production. Hence firms strategically choose positive debt levels which reduces rival’s output level and raise own profits.

The second linkage between output and financial market is the strategic bankruptcy effect. A firm’s susceptibility to financial distress depends on its financial structure, and its fortunes will usually improve if one or more of
its rivals can be driven into financial distress. Therefore, firms might make output market decisions that raise the chances of driving their rivals into insolvency. Since the possibility of financial distress for each firm is contingent on its financial structure, a highly leveraged firm is more vulnerable relative to an unleveraged firm. Therefore the strategic bankruptcy implies that, in the repeated oligopoly game, firms select non-debt financing as the financial decision in order to avoid the predation by rivals.

After the seminal work by Brander and Lewis (1986) on the relationships between financial decisions and output market decisions, many sequential studies have developed their model and shown various results on the linkage between the production equilibrium and the debt selection. Studies by Maksimovic (1988), Stenbacka (1994) and Showalter (1995) are based on the limited liability effect that is a moral hazard between stock-holding managers and creditors. On the other hand, papers by Poitevin (1989) and Bolton and Scharfstein (1990) focus on the predation threat that must be considered in repeated games. In this research, I examine how the limited liability effect and the predation threat affect the production decisions and financial decisions under perfectly correlated demand shocks in a repeated Oligopoly set-up. This approach of focusing on the correlated demand shock shows how results are different from the identical and independent shock case.

I set the three-stage game: At the first stage firms choose financial decision. In the next stage firms select production levels given financing conditions. The leverage firms have the duty to pay debt payments to debt holders. If the
firm’s profit is not sufficient to match the amount it must pay, the firm must go bankrupted. If both firms survive in the second stage, then both play duopoly game in the third stage. If the rival firm fails to repay the debt duty but the firm survives in the previous stage, this firm plays the monopolist in the third stage. Then if one firm issues debt, then the rival firm has an incentive to prey on the leverage firm in the first output game in order to monopolize the next period market (the strategic bankruptcy effect). Hence the leveraged firm is very vulnerable to survive in the first period. On the other hand, the leverage firm has the limited liability that the firm does not lose the whole profit but the stock returns when it is bankrupted (the limited liability effect). Therefore, in this setting, two main relationships between financial decision and production decisions must be considered simultaneously.

Brander and Lewis (1986) assume the independent demand shocks for the simplicity in their model. However, many industrial organization studies such as Green and Porter (1984) and Porter (1983) assume the perfectly correlated shocks as the random terms on the demand. In the case where a firm’s demand shocks are perfectly correlated, the strategic bankruptcy effect cannot occur in a symmetric debt-output equilibrium, because no firm has a chance to be a monopolist in the next period. If there exists a symmetric equilibrium and both firms choose it, then both go bankrupt or survive together. Then each firm has an incentive to deviate the equilibrium and take a positive probability to be monopolist by increasing output level or by decreasing debt level. This is an intuition of no symmetric equilibrium under common shocks. In this
case, there exist multiple equilibria, and firms choose asymmetric equilibrium. This finding is dramatically different from the \( i.i.d \) demand shock case.

I propose a subgame perfect Nash equilibrium with punishment where there exists an optimal debt level supported by firms for any discount factor. The two asymmetric pure output equilibria are used as a punishment against the firm deviating from the optimal debt level in the mixed strategy equilibrium; for the deviating firm, the asymmetric output equilibrium is worse than the mixed output strategy equilibrium. In a repeated game, this punishment supports a symmetric optimal debt level and the mixed output strategy. As in the independent shock case, a positive debt level is selected as the optimal debt level if the present value of future stage gains are very low, but all equity financing is selected if the present value of future stage gains are very high.

The outline of the paper is as follows. Section 2 sets out the basic model. Section 3 is devoted to the output market equilibrium and shows the dependence of output equilibrium on financial structure. Section 4 examines the selection of debt levels and describes the subgame perfect Nash equilibrium with punishment. The last section summarizes results and provides conclusions.

2.2 The Model

The model consists of three stages, a financial decision and two output games. Each firm needs \( I \) as the investment for a business and it must decide
whether issuing debt or stock to finance \( I \). The financial structure of firm \( i \) is defined by \( d_i \), which is the amount of debt obligation firm \( i \) has to pay only at the first period. Firm \( i \) and creditors contract debt covenant which requires that firm \( i \) borrows money from creditors and has to repay \( d_i \) to the creditor at the first period. The debt level is assumed to be chosen before output decision.

If firm \( i \) is unable to meet its debt obligation at the first period, the firm \( i \) should go bankrupt and its bondholders receive residual profits. If firm \( i \) goes bankrupt at the first period, it could not play in the second period output market.

Firms \( i \) and \( j \) play twice repeated game in an output market where they produce competing products \( q_{it} \) and \( q_{jt} \) at each period \( t = 1 \) and \( 2 \), respectively. For concreteness, we assume there is Cournot quantity competition in the output market.

The each period profit for firm \( i \) at \( t \), which is defined as the difference between revenue and variable cost, is defined by \( \Pi_{it}(q_{it}, q_{jt}, \varepsilon_i) \). For simplicity, it is assumed that there exists a random variable \( \varepsilon \) at the first-period, which reflects the effects of an uncertain demand on firm \( i \) at the first-period. Additionally, for simplicity, it is assumed to be uniformly distributed over the interval [0,1].

The important assumption of the Brander and Lewis model is that the random demand shock, \( \varepsilon_i \) is \( i.i.d \) over \( \forall i \neq j: F(\varepsilon_i, \varepsilon_j) = F(\varepsilon_i)F(\varepsilon_j) \). In this paper, we examine how firms decide the financial and output decision,
when the demand shock is completely correlated common shocks over firms: 
$\epsilon_i = \epsilon_j = \epsilon$ is uniformly distributed over $[0, 1]$. For simplicity, we change the model, where firms do not have to meet the debt obligation in the second period. So, in subgames, firms play Cournot output, $q^C$ and monopoly output, $q^M$ and retain $\Pi^C$ and $\Pi^M$ respectively.

We assume that $\Pi_{it}$ satisfies the usual properties:

$$\frac{\partial^2 \Pi_{it}}{\partial q_{it}^2} < 0, \quad \frac{\partial \Pi_{it}}{\partial q_{jt}} < 0, \quad \frac{\partial^2 \Pi_{it}}{\partial q_{it} \partial q_{jt}} < 0$$

(2.2.1)

We adopt the convention that high values of $\epsilon_{it}$ lead to higher operating profits, meaning that higher realizations of $\epsilon_{it}$ correspond to better states of the world.

$$\frac{\partial \Pi_{it}(q_{it}, q_{jt}, \epsilon)}{\partial \epsilon} > 0$$

(2.2.2)

Since two firms play the standard quantity competition, we additionally assume that high values of $\epsilon_{it}$ corresponded to upward shifts in the marginal revenue schedule facing the firm\(^1\):

$$\frac{\partial^2 \Pi_{i1}(q_{it}, q_{jt}, \epsilon)}{\partial q_{it} \partial \epsilon} > 0, \quad \frac{\partial^2 \Pi_{i1}(q_{it}, q_{jt}, \epsilon)}{\partial q_{jt} \partial \epsilon} \leq 0$$

(2.2.3)

If only one of both firms survives and the other is bankrupted in the first period ($t = 1$), then the firm $i$ exists as a monopolist in the second period market ($t = 2$). Then the monopolist $i$ decides the monopoly output level, $q^M_{i2}$ and receives the expected monopoly profits, $\Pi^M_{i2}$ in the second period. If both

\(^1\)Brander and Lewis (1986) explain that this assumption is the standard case under quantity competition, while $\frac{\partial^2 \Pi_{it}(q_{it}, q_{jt}, \epsilon_{it})}{\partial q_{it} \partial \epsilon_{it}} < 0$ may arise when firms engage in other forms of competition besides quantity or price competition.
of firms $i$ and $j$ survive at the first period, then both choose outputs $q^C_{i2}$ and $q^C_{j2}$ at the second output market and receive $\Pi^C_{i2}$ and $\Pi^C_{j2}$.

The first period expected stock return of firm $i$ is defined by

$$S_{i1} = \int_0^1 \max\{\Pi_{i1}(q_{i1}, q_{j1}, \varepsilon) - d_i, 0\} d\varepsilon$$

where $z_i$ is defined by,

$$\Pi_{i1}(q_{i1}, q_{j1}, z_i) - d_i = 0$$

assuming $0 < z_i < 1$. Therefore, the firm $i$ survives at the period 1 with

$$\Pr(\varepsilon > z_i) = 1 - z_i.$$

### 2.3 Output Market Equilibrium

#### 2.3.1 One Shot Game

When the shock is independent, $F(\varepsilon_i, \varepsilon_j) = F(\varepsilon_i)F(\varepsilon_j)$, the first order condition of the firm $i$’s stock return maximization in the one shot game is given by,

$$\frac{dS_i}{dq_i} = \int_{z_i}^1 \frac{\partial \Pi_i(q_i, q_j, \varepsilon_i)}{\partial q_i} f(\varepsilon_i) d\varepsilon_i = 0$$

If the shock is common, i.e. $f(\varepsilon_1, \varepsilon_2) = f(\varepsilon)$, then we get the stock return as follows:

$$S_i = \int_0^1 \max\{\Pi_i(q_i, q_j, \varepsilon) - d_i, 0\} f(\varepsilon) d\varepsilon$$

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where \( z_i \) is defined by \( \Pi_i(q_i, q_j, z_i) - d_i = 0 \) The first order condition of (2.3.2) is given by,
\[
\frac{dS_i}{dq_i} = \int_{z_i}^1 \frac{\partial \Pi_i(q_i, q_j, \varepsilon)}{\partial q_i} f(\varepsilon) d\varepsilon = 0
\]  
(2.3.3)

Two first order conditions, (2.3.1) and (2.3.3) imply that the one shot game output levels, \((q_i, q_j)\) are exactly the same in the independent shock case given debt levels \((d_i, d_j)\).

### 2.3.2 Repeated Game

The probability that firm \( i \) is bankrupted at \( t = 1 \) is denoted by \( \Pr(\varepsilon < z_i) \equiv G(z_i) = z_i \), if \( \varepsilon \) is uniformly distributed over \([0,1]\). Then we have four possibilities:

- Both \((i \text{ and } j)\) solvent: \( \Pr(z_i < \varepsilon \cap z_j < \varepsilon) = \Pr(\max\{z_i, z_j\} \leq \varepsilon) \)
- Only \( i \) monopolist: \( \Pr(z_i < \varepsilon \cap z_j > \varepsilon) = \Pr(z_i < \varepsilon \leq z_j) \)
- Only \( j \) monopolist: \( \Pr(z_i > \varepsilon \cap z_j < \varepsilon) = \Pr(z_j < \varepsilon \leq z_i) \)
- Both \((i \text{ and } j)\) bankrupted: \( \Pr(z_i > \varepsilon \cap z_j > \varepsilon) = \Pr(\min\{z_i, z_j\} > \varepsilon) \)

Since the firm \( i \) is interested in only first two cases among above four probabilities, the present value of stock is given by,
\[
V_i = S_{i1} + \beta [\Pr(z_i < \varepsilon \leq z_j)\Pi^M + \Pr(\max\{z_i, z_j\} \leq \varepsilon)\Pi^C]
\]
Mathematically, the probabilities are given by,

\[ \Pr(z_i < \varepsilon \leq z_j) = \begin{cases} 
z_j - z_i, & \text{if } z_j > z_i \\
0, & \text{otherwise}
\end{cases} \]

\[ \Pr(\max\{z_i, z_j\} \leq \varepsilon) = 1 - \max\{z_i, z_j\} = \begin{cases} 
1 - z_j, & \text{if } z_j > z_i \\
1 - z_i, & \text{otherwise}
\end{cases} \]

Therefore,

\[ V_i = \begin{cases} 
S_i + \beta \left[(z_j - z_i) \Pi^M + (1 - z_j) \Pi^C\right], & \text{if } z_i < z_j \\
S_i + \beta (1 - z_i) \Pi^C, & \text{if } z_i \geq z_j
\end{cases} \quad (2.3.4) \]

Recall that the bankruptcy rate, \( z_i \), is the function of \( q_i, q_j \) and given \( d_i \). If \( z_i < z_j \), then the firm \( i \) has an incentive to maximize \( S_i + \beta [(z_j - z_i) \Pi^M + (1 - z_j) \Pi^C] \). But if \( z_i \geq z_j \), the firm \( i \) has two incentives. The first one is to maximize \( S_i + \beta (1 - z_i) \Pi^C \), which means that the firm \( i \)'s production \( q_i \) is less than BL level given debt levels. The second incentive is to increase \( z_j \) by raising \( q_i \) in order to change to \( z_i < z_j \) from \( z_i \geq z_j \) (because \( \frac{\partial z_i}{\partial q_i} > \frac{\partial z_j}{\partial q_i} \) ), which means that the firm \( i \) increase output more than BL level given debt levels.

Suppose that both firms select the same debt levels, \( d_i = d_j = d \). Then the output decision is very close to “the game of chicken”. If each firm competes to raise its output to make it higher than rival’s, then both of them will face higher bankruptcy rate. If each firm tries to decrease its output less than rival’s output, then it is better for the rival to increase its output. Therefore, note that there is no pure symmetric output strategy with the symmetric debt level under the common shock unlike the independent shock case.
Lemma 2.3.1. Suppose that both firms, \( i \) and \( j \) select the same debt levels, \( d_i = d_j = d \). Then for any \( d > 0 \), there is no pure symmetric output equilibrium.

Proof. Since both firms choose the same debt level, each firm’s bankruptcy rate depends on its output level. If a firm \( i \) chooses the output level \( q_i > q_j \), then its own bankruptcy rate \( z_i \) is lower than the rival’s, \( z_j \). If the firm \( i \) chooses the same (less) output level, \( q_i = q_j \ (q_i < q_j) \), then its own bankruptcy rate, \( z_i = z_j \ (z_i < z_j) \). Suppose that there is a symmetric output equilibrium \((q^0, q^0)\) under a symmetric debt level \( d \). Denote that \( z^0 \equiv z_i(q^0, q^0; d) = z_j(q^0, q^0; d) \). Since both firms choose a symmetric equilibrium, then the firm \( i \)'s whole stock value is as follows: \( V_i = S_i + \beta (1 - z_i) \Pi^C \) and the first order condition is,

\[
\frac{dV_i}{dq_i} = \frac{dS_i}{dq_i} - \beta \left( \frac{dz_i}{dq_i} \right) \Pi^C = 0
\]

If \( q^0 = q^{BL} \), the Brander and Lewis equilibrium given \( d \), the first term of the FOC must be zero but the second term is negative. Then \((q^{BL}, q^{BL})\) could not be the symmetric output equilibrium.

If \( q^0 > q^{BL} \), the first term of FOC is negative as well as the second term is, because of the second order condition and the other condition, \( \frac{d^2 S_i}{dq_i dq_j} < 0 \). Therefore any symmetric output level which are greater than the Brander and Lewis equilibrium could not be the output equilibrium either.

If \( q^0 < q^{BL} \), the first order condition might be satisfied. However this output level is not the best response for each firm as follows:
Suppose \((q^0, q^0)\) satisfies the above FOC. If both firms keep the symmetric output level \(q^0\), the firm \(i\)'s expected stock returns are

\[ V_i(q^0, q^0; d) = S_{i1}(q^0, q^0; d) + \beta(1 - z^0)\Pi^C \]

Denote \(\tilde{q} \equiv q^0 + \eta\) where \(\eta\) is very tiny positive term. If the firm \(i\) takes \(\tilde{q}\) given the firm \(j\)'s output level \(q^0\),

\[ V_i(q^0 + \eta, q^0; d) = S_{i1}(q^0 + \eta, q^0; d) + \beta \left[(z_j - z_i)\Pi^M + (1 - z_j)\Pi^C\right] \]

Note that the payoff function is continuous, if firm \(i\) changes from \(q^0\) to \(\tilde{q}\). Then the difference between two whole stock values is,

\[
\begin{align*}
V_i(q^0 + \eta, q^0; d) &- V_i(q^0, q^0; d) \\
&= S_{i1}(q^0 + \eta, q^0; d) - S_{i1}(q^0, q^0; d) + \beta \left[(z_j - z_i)\Pi^M + (z^0 - z_j)\Pi^C\right] \\
&= S_{i1}(q^0 + \eta, q^0; d) - S_{i1}(q^0, q^0; d) \\
&\quad + \beta \left[(z_j - z^0 - z_i + z^0)\Pi^M - (z^0 - z_j)\Pi^C\right]
\end{align*}
\]

Then,

\[
\begin{align*}
\lim_{\eta \to 0^+} \frac{V_i(q^0 + \eta, q^0; d) - V_i(q^0, q^0; d)}{\eta} &= \lim_{\eta \to 0^+} \frac{S_{i1}(q^0 + \eta, q^0; d) - S_{i1}(q^0, q^0; d)}{\eta} \\
&\quad + \lim_{\eta \to 0^+} \frac{\beta \left[(z_j - z^0 - z_i + z^0)\Pi^M - (z^0 - z_j)\Pi^C\right]}{\eta} \\
&> 0
\end{align*}
\]

The first term is the positive, because of \(\frac{d^2 S_i}{dq_0 dq_j} < 0\) and \(\frac{d^2 S_i}{d^2 q_i} < 0\). Since \(\frac{d^2 S_{iBL}}{dq_i dq_j} = 0\), the derivative \(\frac{dS_i}{dq_i}\) around \((q^0, q^0)\) is positive when \(q^0 < q^{BL}\).
Since $\eta$ is a very tiny positive term, the second part of above equation is positive as well\(^2\). Therefore, each firm can increase its own stock value through raising its output level. Hence no one keeps this pure symmetric output levels $(q^0, q^0)$. Q.E.D.

**Lemma 2.3.2.** There are two asymmetric pure output strategies given a symmetric debt level $d > 0$ under the common shock.

**Proof.** There is an asymmetric output equilibrium, $(q^H, q^L)$ where $q^H > q^L$. The first order condition for the firm $i$ is,

$$
\frac{dV_i}{dq_i} = \frac{dS_{i1}}{dq_i} + \beta \left[ \left( \frac{dz_j}{dq_i} - \frac{dz_i}{dq_i} \right) \Pi^M - \frac{dz_j}{dq_i} \Pi^C \right] = 0
$$

(2.3.5)

$$
\Leftrightarrow \frac{dS_{i1}}{dq_i} + \beta \left[ \frac{d}{q_i^3} \Pi^M - b \Pi^C \right] = 0
$$

and the FOC for $j$ is,

$$
\frac{dV_j}{dq_j} = \frac{dS_{j1}}{dq_j} - \beta \frac{dz_i}{dq_j} \Pi^C = 0
$$

(2.3.6)

$$
\Leftrightarrow \frac{dS_{j1}}{dq_j} - \beta \left( - \frac{d}{q_j^3} + b \right) \Pi^C = 0
$$

The SOC is,

$$
\frac{d^2V_i}{dq_i^2} = \frac{d^2S_{i1}}{dq_i^2} + \beta \left[ \frac{d^2z_j}{dq_i^2} (\Pi^M - \Pi^C) - \frac{d^2z_i}{dq_i^2} \Pi^M \right] = \frac{d^2S_{i1}}{dq_i^2} - \beta \left( \frac{d}{q_i^3} \right) \Pi^M < 0
$$

(2.3.7)

\(^2\)Because the minimum level of output is $\sqrt{\frac{d}{b}}$ and the maximum level is $\sqrt{\frac{d}{\frac{d}{q_i^3} \Pi^M}}$.  

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\[
\frac{d^2 V_j}{d^2 q_j} = \frac{d^2 S_{j1}}{d^2 q_j} - \beta \frac{d^2 z_i}{d^2 q_j} \Pi^C = \frac{d^2 S_{j1}}{d^2 q_j} - \beta \left( \frac{d}{q_j^3} \right) \Pi^C < 0 \quad (2.3.8)
\]

The cross derivatives are,
\[
\frac{d^2 V_i}{dq_i dq_j} = \frac{d^2 S_{i1}}{dq_i dq_j} + \beta \left[ \frac{d^2 z_j}{dq_i dq_j} \left( \Pi^M - \Pi^C \right) - \frac{d^2 z_i}{dq_i dq_j} \Pi^M \right] = \frac{d^2 S_{i1}}{dq_i dq_j} < 0 \quad (2.3.9)
\]
\[
\frac{d^2 V_j}{dq_j dq_i} = \frac{d^2 S_{j1}}{dq_j dq_i} - \beta \frac{d^2 z_j}{dq_j dq_i} \Pi^C = \frac{d^2 S_{j1}}{dq_j dq_i} < 0 \quad (2.3.10)
\]

Then,
\[
\frac{d^2 V_i}{d^2 q_i} \frac{d^2 V_j}{d^2 q_j} - \frac{d^2 V_i}{dq_i dq_j} \frac{d^2 V_j}{dq_j dq_i} > 0 \quad (2.3.11)
\]

Therefore, the asymmetric output pair, \((q^H, q^L)\) satisfying two FOCs is stable and one equilibrium given the debt level, \(d\).

Similarly, we can show that reverse quantities, \((q^L, q^H)\) is the other equilibrium. Q.E.D. \(\Box\)

**Lemma 2.3.3.** Assume firms \(i\) and \(j\) are symmetric. Then the asymmetric output equilibrium \(q^H\) and \(q^L\) are increasing in the symmetric debt level \(d\).

**Proof.** From the FOCs at \((q^H, q^L)\),
\[
\frac{d^2 V_i}{dq_i dd} = \frac{d^2 S_i}{dq_i dd} + \beta \left( \frac{1}{q^2_i} \right) \Pi^M > 0 \quad (2.3.12)
\]
\[
\frac{d^2 V_j}{dq_j dd} = \frac{d^2 S_j}{dq_j dd} + \beta \left( \frac{1}{q^2_j} \right) \Pi^C > 0
\]

Then,
\[
\frac{dq^H}{dd} = -\frac{d^2 V_i}{dq_i dd} \frac{d^2 V_i}{d^2 q_i} > 0 \quad (2.3.13)
\]
\[
\frac{dq^L}{dd} = -\frac{d^2 V_j}{dq_j dd} \frac{d^2 V_j}{d^2 q_j} > 0
\]
Therefore, note that non-leverage financing is the best choice to both firms, if it is supported by two identical firms given the discount factor \( \beta \). Additionally, note that the an asymmetric output equilibrium, \( q^H \) is increasing in the discount factor \( \beta \) and \( q^L \) is decreasing in \( \beta \), because, from the FOCs,

\[
\frac{d^2V_i}{dq_id\beta} = \left( \frac{dz_j}{dq_i} - \frac{dz_i}{dq_i} \right) \Pi^M - \frac{dz_j}{dq_i} \Pi^C > 0
\]

\[
\frac{d^2V_j}{dq_jd\beta} = -\frac{dz_j}{dq_j} \Pi^C < 0
\]

Then,

\[
\frac{dq^H}{d\beta} = -\frac{d^2V_i}{dq_id\beta} \frac{d^2V_i}{d^2q_i} > 0
\] (2.3.14)

\[
\frac{dq^L}{d\beta} = -\frac{d^2V_j}{dq_jd\beta} \frac{d^2V_j}{d^2q_j} < 0
\]

Additionally, there exists the mixed strategy beyond two asymmetric pure equilibria as Dasgupta and Maskin (1986a) show the existence of mixed strategy equilibrium in games with discontinuous payoff functions\(^3\).

When the debt levels are differently selected, the similar logic is applied. The probability of the monopolist in the second period is determined by the two rates \( d_i/q_i \) and \( d_j/q_j \). These two rates determines the size of \( z_i \) and \( z_j \). In

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\(^3\)See Theorems 4 and 5 in Dasgupta and Maskin (1986a).
the BL equilibrium, if $d_i < d_j$, the bankruptcy rate of firm $i$, $z_i$ is less than
the firm $j$’s $z_j$.

Assuming the gap between two debt levels satisfies the conditions where
the high leveraged firm is not preyed on by the low leveraged firm, we can
show the existence of two asymmetric pure output equilibria. Since the size
of $z_i$ and $z_j$ determines the two rates $d_i/q_i$ and $d_j/q_j$ of firm $i$ and firm $j$,
the output levels $(q_i, q_j)$ satisfying $d_i/q_i = d_j/q_j$ cannot be the equilibrium
similarly under $(d_i, d_j)$ as shown in lemma 2.3.1.

**Lemma 2.3.4.** Suppose that the gap between two different debt levels satisfies
the conditions where the high leveraged firm could not be preyed on by the low
leveraged firm. Then there are two pure output equilibria.

**Proof.** The proof of this lemma is similar to proofs of the lemmas 2.3.1 and
2.3.2. See Appendix B.1.

As the same as in the symmetric debt level case, there exists the mixed
strategy beyond two asymmetric pure equilibria as Dasgupta and Maskin

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4In the linear case, the condition is that $(d_i, d_j)$ belongs to the set,

$$\{(d_i, d_j) | [a + 1 - b(1 + d_i/d_j)q_i^{BL}(d_i, d_j)] q_i^{BL}(d_i, d_j) \leq d_i \}$$

If this condition is not satisfied (i.e. if the gap between two firms debt levels is huge enough
to deviate from above set), in order to avoid the predation by the lower leveraged firm the
higher leveraged firm has to choose the lower output equilibirum which is the unique output
strategy under different debt levels.
(1986a) show the existence of mixed strategy equilibrium in games with discontinuous payoff functions in the different debt levels.

2.4 Financial Equilibrium

In this subsection, we consider a subgame Nash equilibrium (SPNE) with punishment as the financial equilibrium. Since both firms choose their output levels after observing rival’s financial decision, then the output strategy is the reaction to selected debt levels.

2.4.1 Punishment Scheme

I suppose a public information in the equilibrium if the symmetric debt level is chosen by two firms. Under the symmetric debt level, suppose each firm chooses $q^H$ and $q^L$ with the probability $1/2$ responding the public signal. This public randomization equilibrium is denoted by $\gamma(d, d)$:

$$
\gamma(d, d) = \begin{cases} 
q^H & \text{with } 1/2 \\
q^L & \text{with } 1/2
\end{cases}
$$

Then, given $(d_i, d_j)$, I consider three output equilibria: two asymmetric equilibria $(q_i^H(d_i, d_j), q_i^L(d_i, d_j)), (q_j^L(d_i, d_j), q_j^H(d_i, d_j))$ and the public randomization $(\gamma(d, d), \gamma(d, d))$.

Suppose both firm want to select $d^*$ as an optimal debt level. Set the symmetric debt level $d_i = d^*$. If $d_j = d^*$, then firm $i$ chooses the mixed strategy $(\gamma(d^*, d^*), \gamma(d^*, d^*))$. If $d_j \neq d^*$, however, the firm $i$ sets $q_i = q_i^H(d_i, d_j)$, the
aggressive output level. Symmetrically, I set the same scheme for the firm $j$.

Then the reaction to the financial decisions are as follows:

$$q_i(d_i, d_j) = \begin{cases} 
q_i^L(d_i, d_j), & \text{if } d_i \neq d^*, d_j = d^* \\
\gamma(d^*, d^*), & \text{if } d_i = d^*, d_j = d^* \\
q_i^H(d_i, d_j), & \text{if } d_i = d^*, d_j \neq d^* 
\end{cases}$$

$$q_j(d_i, d_j) = \begin{cases} 
q_j^L(d_i, d_j), & \text{if } d_i = d^*, d_j \neq d^* \\
\gamma(d^*, d^*), & \text{if } d_i = d^*, d_j = d^* \\
q_j^H(d_i, d_j), & \text{if } d_i \neq d^*, d_j = d^* 
\end{cases}$$

If the firm $j$ deviates $d^*$ and increases as much as $\Delta > 0$, then output strategy is changed from $(\gamma(d^*, d^*), \gamma(d^*, d^*))$ to $(q_i^H(d^*, d^* + \Delta), q_j^L(d^*, d^* + \Delta))$.

Then the change in the total stock returns of the firm $j$, $\Delta V_j$ is,

$$\Delta V_j \equiv V_j((q_i^H(d^*, d^* + \Delta), q_j^L(d^*, d^* + \Delta)) - V_j((\gamma(d^*, d^*), \gamma(d^*, d^*))) \tag{2.3.15}$$

$$= [V_j((q_i^H(d^*, d^* + \Delta), q_j^L(d^*, d^* + \Delta)) - V_j((q_i^H(d^*, d^*), q_j^L(d^*, d^*))]
+ [V_j((q_i^H(d^*, d^*), q_j^L(d^*, d^*)) - V_j((\gamma(d^*, d^*), \gamma(d^*, d^*)))]$$

and the change in the firm $j$’s debt return, $\Delta W_j$ is,

$$\Delta W_j \equiv W_j((q_i^H(d^*, d^* + \Delta), q_j^L(d^*, d^* + \Delta)) - W_j((\gamma(d^*, d^*), \gamma(d^*, d^*))) \tag{2.3.16}$$

$$= [W_j((q_i^H(d^*, d^* + \Delta), q_j^L(d^*, d^* + \Delta)) - W_j((q_i^H(d^*, d^*), q_j^L(d^*, d^*))]
+ [W_j((q_i^H(d^*, d^*), q_j^L(d^*, d^*)) - W_j((\gamma(d^*, d^*), \gamma(d^*, d^*)))]$$

where $W_j(q_i, q_j) = \int_0^{z_j} [\Pi_j(q_i, q_j, \varepsilon) - d_j] d\varepsilon$ and $z_j$ s.t. $\Pi_j(q_i, q_j, z_j) - d_j = 0$. 

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Therefore, the change in the whole present value of firm \( j \), \( \Delta Y_j \) is,

\[
\Delta Y_j = \Delta V_j + \Delta W_j
\]

\[
= [Y_j(\{q^H_i(d^*, d^* + \Delta), q^L_i(d^*, d^*)\}) - Y_j(\{q^H_i(d^*, d^*), q^L_i(d^*, d^*)\})]
\]

Since each firm selects the debt level which maximizes the whole present value as shown in the section 3.2, both firm deviate from \( d^* \) if \( \Delta Y_j \) is positive. Hence if there exists any positive change \( \Delta \), then \( d^* \) cannot be selected anymore. The first bracket of equation (2.3.17) represents the limited liability effect on the penalty and the second one means the penalty of deviation. Then denote that the limited liability effect (LLE) and the penalty to preventing deviation effect (PE) as follows:

\[
LLE(d^*, \beta) \equiv [Y_j(\{q^H_i(d^*, d^* + \Delta), q^L_i(d^*, d^*)\}) - Y_j(\{q^H_i(d^*, d^*), q^L_i(d^*, d^*)\})]
\]

\[
PE(d^*, \beta) \equiv [Y_j(\{\gamma(d^*, d^*), \gamma(d^*, d^*)\}) - Y_j(\{q^H_i(d^*, d^*), q^L_i(d^*, d^*)\})]
\]

(2.3.18)

(2.3.19)

The penalty effect (PE) is increasing in the discount factor \( \beta \), because \( \frac{dL^H}{d\beta} > 0 \) and \( \frac{d\beta}{d\beta} < 0 \). Since \( q^H(d^*, d^*) = q^L(d^*, d^*) = \gamma(d^*, d^*) \) at \( \beta = 0 \), for all \( \beta > 0 \) PE is positive\(^5\). Therefore, as \( \beta \) rises, PE becomes huger,

\(^5\)The PE is given by

\[
\frac{1}{4} Y_j(q^H_i(d^*, d^*), q^H_i(d^*, d^*)) + \frac{1}{4} Y_j(q^L_i(d^*, d^*), q^L_i(d^*, d^*))
\]

\[
+ \frac{1}{4} Y_j(q^L_i(d^*, d^*), q^H_i(d^*, d^*)) - \frac{3}{4} Y_j(q^H_i(d^*, d^*), q^L_i(d^*, d^*))
\]

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while PE is zero at $\beta = 0$. But since the lemma 2.3.3, the change in $d^*$ affects less PE than does the discount factor $\beta^6$.

On the other hand, LLE is affected by the change in $d^*$. Suppose that $\Delta$ is very close to zero, then we can consider the derivative of LLE on the penalty$^7$:

$$
\left[ \frac{dY_j(q_i^H, q_i^L)}{dd_j} \right]_{(d^*,d^*)} = \left[ \frac{dq_i^L}{dd_j} \right] \left\{ \int_0^{z_j} \frac{\partial \Pi_j}{\partial q_j} dz \right\} + \left[ \frac{dq_i^H}{dd_j} \right] \left\{ \int_0^1 \frac{\partial \Pi_j}{\partial q_i} dz \right\} \\
- \beta \left\{ \left( \frac{dz_j}{dq_i^H} \right) \left( \frac{dq_i^H}{dd_j} \right) + \left( \frac{dz_j}{dq_i^L} \right) \left( \frac{dq_i^L}{dd_j} \right) \right\} \Pi^C
$$

$^6$The effect of increases in the select debt $d^*$ on PE is given by,

$$
\frac{dPE}{dd^*} = \frac{1}{2} \left( \frac{dY_j}{dd^*} - \frac{dY_i}{dd^*} \right) \left( \frac{dq_i^H}{dd^*} - \frac{dq_i^L}{dd^*} \right)
$$

Since lemma 2.3.3., the sign of above expression is mathematically ambiguous but this comparative statics is very close to zero relative to one of LLE as shown below.

$^7$See Appendix B.2.
where
\[
\begin{align*}
\left[ \frac{dq_j^L}{dd_j} \right] &= - \left( \frac{d^2V_j}{dq_j^L dd_j} \right) \left( \frac{d^2V_i}{d^2q_i^L} \right) / G > 0 \\
\left[ \frac{dq_j^H}{dd_j} \right] &= \left( \frac{d^2V_j}{dq_j^H dd_j} \right) \left( \frac{d^2V_i}{d^2q_i^H} \right) / G < 0
\end{align*}
\]
and
\[
G = \frac{d^2V_i d^2V_j}{d^2q_i^H d^2q_j^L} - \frac{d^2V_i}{d^2q_i^H dq_j^L} \frac{d^2V_j}{d^2q_j^L dq_i^H}
\]

While \( \left[ \frac{dq_j^L}{dd_j} \right] \) is positive and \( \left[ \frac{dq_j^H}{dd_j} \right] \) is negative, \( \left\{ \left( \frac{dz_j}{dq_j^H} \right) \left[ \frac{dq_j^H}{dd_j} \right] + \left( \frac{dz_j}{dd_j} \right) \right\} \) is positive in the linear demand function. Therefore, the limited liability effect, \( \left[ \frac{dY_j(q_i^H, q_j^L)}{dd_j} \right] \) is decreasing in \( \beta \). When the discount factor is zero (the BL model), then the LLE must be positive at zero debt, \( d^* = 0 \). The size of LLE is given by,
\[
LLE(d^*, \beta) = \int_{d^*}^{d^* + \Delta} \left\{ \frac{dY_j(q_i^H, q_j^L)}{dd_j} \right\} dd_j \quad (2.3.21)
\]

Then the LLE is decreasing in the discount factor \( \beta \) for a certain \( \Delta \) given \( d^* \).

At \( \beta = 0 \), there must be \( \Delta > 0 \) such that the LLE is positive and \( \Delta Y_j \) is also positive, because the PE is zero at \( \beta = 0 \).

Note that \( \left[ \frac{dY_j(q_i^H, q_j^L)}{dd_j} \right] \) is decreasing in the optimal debt \( d^* \). Since lemma 2.3.3., increases in \( d^* \) raise \( q_i^H \) and \( q_j^L \) simultaneously and then the bankruptcy rate of the firm \( j \), \( z_j \) rises. Then \( \int_0^{z_j} \frac{dY_j}{d\varnothing} d\varnothing \) increases so that the derivative decreases, because \( \left[ \frac{dq_j^H}{dd_j} \right] \) is negative. Therefore, \( \left[ \frac{dY_j(q_i^H, q_j^L)}{dd_j} \right] \) is decreasing in the optimal debt \( d^* \). Hence the LLE is decreasing in \( d^* \) for a certain \( \Delta \) given \( \beta \).

If the deviation \( \Delta \) is close to zero, LLE is close to zero, too. As the derivative \( \Delta \) rises, the expression (3.2.20) decreases, because as debt level increases.
the bankruptcy rate rises and so the first term in (2.3.20) falls. Therefore, the deviating firm wants to choose $\Delta$ which induces \( \frac{dY_j(q_i^H, q_i^L)}{dd_j} \bigg|_{(d^*, d^*+\Delta)} = 0 \) to maximize LLE. Denote the deviation $\Delta$ which maximizes LLE given $d^*$ and $\beta$ by

\[
\Delta^{Max}(d^*, \beta) \equiv \arg \max_\Delta \int_{d^*}^{d^*+\Delta} \left\{ \frac{dY_j(q_i^H, q_i^L)}{dd_j} \right\} dd_j \text{ given } d^* \text{ and } \beta \quad (2.3.22)
\]

and denote the maximum LLE,

\[
LLE(d^*, \beta|\Delta^{Max}) \equiv \int_{d^*}^{d^*+\Delta^{Max}} \left\{ \frac{dY_j(q_i^H, q_i^L)}{dd_j} \right\} dd_j \text{ given } d^* \text{ and } \beta \quad (2.3.23)
\]

Since the derivative \( \left[ \frac{dY_j(q_i^H, q_i^L)}{dd_j} \right] \bigg|_{(d^*, d^*)} \) is decreasing in the discount factor, $\Delta^{Max}$ is decreasing in $\beta$ given $d^*$, too. Therefore, it means the maximum LLE, $LLE(d^*, \beta|\Delta^{Max})$ is also decreasing in $\beta$ given $d^*$ like LLE. Figure 2.1 shows that the maximum LLE and the PE. As shown above the PE is zero at $\beta = 0$ and has a positive slope over all $\beta$ while the maximum LLE is positive at $\beta = 0$ and has a negative slope over all $\beta$ if the selected debt level is zero. Suppose that zero debt level is chosen as the optimal debt level $d^*$. Then there exists the discount factor $\underline{\beta}$ at which PE and the maximum LLE are equal and so $\Delta Y_j$ is zero as shown in Figure 2.1.

When the discount factor is $\underline{\beta}$, if non-leverage financing is selected as the optimal debt level, then no firm wants to hold debt and two firms choose Cournot equilibrium in each output market. However, the optimal debt $d^* = 0$ cannot be supported for all $\beta < \underline{\beta}$. Because there is a positive $\Delta^{Max}$ which makes the the maximum LLE is greater than the PE for $\beta < \underline{\beta}$. For example,
at $\beta^*$, each firm has an incentive to issue debt in order to improve its whole present value, because the maximum LLE is bigger than the PE given $d^* = 0$.

If the size of PE is bigger than the maximum LLE’s, the deviation from $d^*$ to $d^* + \Delta$ reduces not only the chance to succeed monopolization but also the first period profit, and so no firm deviates the selected level. However if the maximum LLE is huger than the PE, the deviation from the symmetric level reduces only the monopoly chance in the next period but raises the current profits.

2.4.2 Selection of Optimal Debt Level

The best selection of debt level is definitely zero, because there is no limited liability effect, and so two firms avoid the prisoner’s dilemma of the
quantity increasing. However, at very low discount factors, zero debt level cannot be supported by the both firms, because the limited liability effect is very high relative to the penalty effect as shown in the example of $\beta^o$ in Figure 2.1. Therefore, at very low discount factors, two firm must allow some positive debt as the optimal debt level.

As the discount factor approaches zero, the discounted second period profits relative to the first period’s become less important, and so the limited liability effect increases while the predation is less worried. Then finally, in the BL equilibrium (where $\beta = 0$), only limited liability effect remains. so that both firms choose a positive debt financing.

Since the asymmetric output equilibrium $q^H$ and $q^L$ strictly increases as the symmetric debt level rises, the optimal debt level must be close to zero given the discount factor. For all discount factor less than $\beta$, however, zero debt financing could not be supported. Since the LLE is decreasing function on the optimal debt level, the gap between the maximum LLE and PE can be diminished as $d^*$ rises\(^8\).

Therefore, for any $\beta < \beta$, we can find the minimum positive debt level at which no firm has an incentive to increase the debt level more than the debt level. For example, for $\beta^o$ there exists $d^*$ at which the maximum LLE and the PE are equal as shown in Figure 2.2 and so no firm deviates $d^*$\(^9\). When $\beta = 0$,

---

\(^8\)The effect of the change in $d^*$ on the PE is ambiguous as mentioned above, and this changing is close to zero relative to the change in the maximum LLE.

\(^9\)At $\beta^o$ $d^*$ satisfies that

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of course, the optimal debt level \( d^* \) is the exactly the BL debt equilibrium, \( d^{BL} \) that must be the maximum optimal debt level over all discount factors. Hence, for \( \beta < \beta_\ast \), the optimal debt level decreases as the discount factor rises. For all \( \beta > \beta_\ast \), the zero debt equilibrium is supported as the optimal debt selection.

Without the penalty scheme to prevent the deviation, the zero debt financing cannot be supported even at \( \beta_\ast \), because the maximum LLE is positive there and so \( \Delta Y_j \) is positive. Through the penalty scheme firms are able to select the lower optimal debt level. Therefore, the optimal debt selection with

\[
\forall \Delta > 0, \int_{d^0}^{d^*+\Delta} \left\{ \frac{dY_j(q_j^H, q_j^L)}{dd_j} \right\} dd_j \leq \begin{bmatrix}
\frac{1}{4}Y_j(q_j^H(d^0, d^\circ), q_j^H(d^0, d^\circ)) \\
+\frac{1}{4}Y_j(q_j^L(d^0, d^\circ), q_j^L(d^0, d^\circ)) \\
+\frac{1}{4}Y_j(q_j^L(d^0, d^\circ), q_j^H(d^0, d^\circ)) \\
-\frac{1}{4}Y_j(q_j^H(d^0, d^\circ), q_j^L(d^0, d^\circ))
\end{bmatrix}
\]

where \( = \) holds if \( \Delta = \Delta^{MAX} \).
the penalty code is better to both firms.

2.5 Conclusions

When duopoly firms’ demand shocks are perfectly correlated, if both firms choose the same debt-output levels, then there are only two possibilities; both survive or both go bankrupted. Hence the strategic bankruptcy effect cannot occur in a symmetric debt-output level unlike in the independent demand shock case, because no firm has a chance to be a monopolist in the next period. Therefore, any symmetric debt-output levels cannot be the equilibrium under the common shocks, because each firm wants to deviate from such a symmetric level and increase output level or decrease debt level to hold the chance to be the monopolist in the second-period output market.

If both firms compete increasing output levels in order to hold the chance to be the monopolist, however, they will lose their own survival rates. Therefore, under the symmetric debt levels, one firm has to accommodate the rival’s high output level and choose low output level, and vice versa. This is the same concept as the game of chickens; the equilibrium is that one is passive and the other is aggressive. Hence there exist two asymmetric output strategies and the mixed strategy under the symmetric debt level.

In the subgame perfect Nash equilibrium with punishment, the optimal debt level is supported by firms for any discount factor. The two asymmetric pure output equilibria are used as a punishment against the firm deviating
from the optimal debt level in the mixed strategy equilibrium: for the deviating firm, the asymmetric output equilibrium is worse than the mixed output strategy equilibrium. In three-stage game, this punishment supports a symmetric optimal debt level and the mixed output strategy. This punishment scheme can reduce each firm’s incentive to increase the output level (the limited liability). Of course, the best debt level is definitely zero, however the zero debt level is not supported at very low discount factor, because the limited liability is huge enough to be greater than the punishment. Therefore, the optimal debt level is chosen as low as possible given the discount factor. Like in the independent shock case, positive debt levels are selected as the optimal debt levels in very low discount factors while zero debt financing is selected in high discount levels. When the discount factor is zero, the optimal debt level is the same as the Brander and Lewis debt equilibrium, because there does not exist the penalty effect any more.
Chapter 3

Oligopoly and Financial Structure:
Effect of Risk Aversion

3.1 Introduction

The research work by Brander and Lewis (1986) studies the linkage between the financial and production decisions under an uncertainty environment, and they show that the leveraged firm produces more than does the non-leveraged firm. This finding is called the limited liability effect which is a sort of moral hazard between creditors and borrowers. As Brander and Lewis (1986) explain, as firms take on more debt, they will have an incentive to pursue output strategies that raise returns in good states and lower returns in bad states. The basic point is that shareholders ignore reductions in returns in bankruptcy states, since bondholders become the residual claimants. As debt levels change, the distribution of returns to shareholders over the different states changes, which in turn changes the output strategy favored by shareholding managers. Therefore, a high leveraged firm concentrating on good states selects a more aggressive production and it commits an unleveraged rival to a passive production.

Their study shows the clear explanation on the manager’s the produc-
tion decision of the leveraged firm under uncertainty. However, their approach assumes the risk neutral in the manager’s decision, and the risk aversion case of the firm’s manager has not been considered. Unless managers are risk indifferent, an uncertain operating environment is likely to have important effects on a manager’s decision. In this research, I examine the production decision of a risk averse manager in the same structure capturing essential aspects of Brander and Lewis model. My study focuses on how the risk aversion affects the limited liability effect under uncertainty. Sequentially, many studies by Maksimovic (1988), McAndrews and Nakamura (1992), Stenbacka (1994) and Showalter (1995) develop the idea of the limited liability in their models, following Brander and Lewis’ (1986) seminal work in the relationship between oligopoly and capital structure. Setting infinitely repeated games, Maksimovic (1988) and Stenbacka (1994) show that the collusion of leveraged firms is more vulnerable and less stable than is one of non-leveraged firms. McAndrews and Nakamura (1992) explain that, since the incumbent in an industry has the first-mover advantage in the financial decision, it strategically chooses debt financing and increases the output level when the new firm enters in the market. The aggressive output decision lowers the entrant profits and deters the new entry. Showalter (1995) points out that an increase in the debt level causes an increase in the price, when uncertainty exists on the demand in the price competition model. Although these studies show various results based on the limited liability effect, any existing model has not considered the effect of risk aversion on the manager’s decision yet.
I use the ordinal properties of the risk aversion in the production decision studied by Sandmo (1971), Leland (1972) and Katz (1983). At first, I show the difference of production decisions between risk averse and neutral managers. Sandmo (1971) points out that the risk averse manager chooses lower output levels than does the risk neutral manager under the uncertainty, because the risk averter overweights low demand states and underweights high demand states. This reasoning also works in the risk averse case so that the stock-holding risk averse manager selects lower production level. One difference from a non-leveraged firm case is that the risk averse manager of the leveraged firm chooses lower output levels, and so his firm has lower bankruptcy rate relative to the risk neutral manager’s firm. It means that the risk averter should consider wider random interval which includes worse states.

At second, I test whether the limited liability effect works in the risk averse manager case. It is true that the stock-holding risk averse manager concerns the bankrupt more than does the risk neutraler. But, as the debt rises, the risk averter also focuses on better states so that the manager increases production responding higher demands. That is, the limited liability effect by Brander and Lewis appears in the risk averse manger case.

Finally, I examine how the degree of risk aversion affects the limited liability effect. In particular, one might come to believe that as the degree of risk aversion rises, the limited liability effect falls. However, this note theoretically shows ambiguous results. There are two points which determine the difference of limited liability effect between risk aversers and risk neutralers.
The first point is that, if the risk averter changes the output levels, it affects not only the marginal stock returns but also the marginal utility. Therefore, this point implies that, as debt rises, the risk averse manager increases less output level than does the risk neutraler. But more important point considered is the second point: as debt rises, the marginal utility becomes lower over all solvency states relative to the highest marginal utility at the lowest solvency state. Then, the risk manager has an incentive to raise output level higher than does the risk neutral manager. The second point indicates that the limited liability effect rises as the degree of risk aversion increases. Therefore, these two points make the results ambiguous, because the comparison of the limited liability effect depends on the summation of these two points.

The outline of the paper is as follows. Section 2 sets out the production market equilibrium. Additionally this section shows the limited liability effect still works in the risk averse manager case. Section 3 compares the limited liability effect between the risk averter and the risk neutraler. The simulation results report how the degree of risk aversion affects the limited liability effect.

3.2 Production Decision

For the simplicity, I use the simple monopoly model of Brander and Lewis (1986) without loss of generality. I assume that the profit function \( \Pi \) satisfies the usual properties:

\[
\frac{\partial^2 \Pi}{\partial^2 q} < 0 \quad (3.2.1)
\]
I adopt the convention that higher values of \( \varepsilon \) lead to higher operating profits, meaning that higher realizations of \( \varepsilon \) correspond to better states of the world.

\[
\frac{\partial \Pi(q, \varepsilon)}{\partial \varepsilon} > 0 \tag{3.2.2}
\]

Since the two firms play in a standard quantity competition, I additionally assume that higher values of \( \varepsilon \) correspond to upward shifts in the marginal revenue schedule facing the firms;

\[
\frac{\partial^2 \Pi(q, \varepsilon)}{\partial q \partial \varepsilon} > 0 \tag{3.2.3}
\]

I assume that \( \varepsilon \) is uniformly distributed between 0 and 1. Suppose the managers have the utility function, \( U(\cdot) \). This utility function satisfies following concavity properties;

\[
U' > 0 \tag{3.2.4}
\]

\[
U'' < 0 \tag{3.2.5}
\]

Then, in particular, the utility function of the risk neutral managers is given by,

\[
U' = 1 \text{ and } U'' = 0 \tag{3.2.6}
\]

As a given condition, I assume

\[
U'(0) = U'(\cdot) < \infty \tag{3.2.7}
\]

In the risk neutral stockholding-manager case, the second order derivatives of the utility function is not negative but positive, and so the payoff
amount, $V$ is given by,

$$V = \int_0^1 \max\{\Pi(q, \varepsilon) - D, 0\} d\varepsilon$$

$$= \int_z^1 \{\Pi(q, \varepsilon) - D\} d\varepsilon$$

where $z$ is defined by

$$\Pi(q, z) - D = 0$$

(3.2.9)

The value to the risk averse share-holding manager is referred to as the equity value is represented by the letter $W$:

$$W = \int_0^1 U(\max\{\Pi(q, \varepsilon) - D, 0\}) d\varepsilon$$

$$= \int_0^z U(0) d\varepsilon + \int_z^1 U(\Pi(q, \varepsilon) - D) d\varepsilon$$

where $z$ is defined by (3.2.9).

The expression (3.2.9) shows the implicit dependence of $z$ on $D$ and $q$.

It is useful to report the following derivatives:

$$\frac{dz}{dD} = \frac{1}{\left(\frac{\partial \Pi(z)}{\partial \varepsilon}\right)} > 0$$

(3.2.11.1)

$$\frac{dz}{dq} = -\frac{\left(\frac{\partial \Pi(z)}{\partial q}\right)}{\left(\frac{\partial \Pi(z)}{\partial \varepsilon}\right)}$$

(3.2.11.2)

This relationship is established in the risk averse manager case as well as in the risk neutral manager case.

The natural assumption, abstracting from agency problems between managers and shareholders, is that managers maximize equity value in this
stage of the game when debt levels are taken as given. Once the debtholders are captive, the managers have no subsequent incentive to act in the debtholders interests.

3.2.1 Risk Neutral Managers

At first, suppose the risk neutral manager case, as a benchmark of the output equilibrium. Assuming an interior solution, the choice of output for the risk neutraler’s firm is obtained by setting the derivative of (3.2.8) with respect to $q$ equal to zero.

$$\frac{\partial V}{\partial q} = \int_z^1 \frac{\partial \Pi(q, \varepsilon)}{\partial q} d\varepsilon = 0 \quad (3.2.12)$$

The second order condition is

$$\frac{\partial^2 V}{\partial^2 q} = \int_z^1 \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial^2 q} \right) d\varepsilon - \left( \frac{\partial \Pi(q, z)}{\partial q} \right) \frac{dz}{dq} < 0 \quad (3.2.13)$$

The the output level that satisfies (3.2.12) is denoted by $q^N$ and the bankruptcy rate responding to $q^N$ is denoted by $z^N$. Therefore, I get

$$\frac{\partial V}{\partial q} = \int_{z^N}^1 \frac{\partial \Pi(q^N, \varepsilon)}{\partial q} d\varepsilon = 0 \quad (3.2.12.1)$$

3.2.2 Risk Averse Managers

Proceeding as before, I characterize the Nash equilibrium in the output market, where now managers act to maximize their value as given by (3.2.10).
Assuming an interior solution, the choice of output for the risk averse’s firm $i$ is obtained by setting the derivative of (3.2.10) with respect to $q$ equal to zero.

$$
\frac{\partial W}{\partial q} = U(0) \frac{dz}{dq} + \int_z^1 U'(\Pi(q, \varepsilon) - D) \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right) d\varepsilon - U(0) \frac{dz}{dq}
$$

$$
\Leftrightarrow \frac{\partial W}{\partial q} = \int_z^1 U'(\Pi(q, \varepsilon) - D) \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right) d\varepsilon = 0 \quad (3.2.14)
$$

The second order condition is

$$
\frac{\partial^2 W}{\partial^2 q} = \int_z^1 U''(\Pi(q, \varepsilon) - D) \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right) d\varepsilon \quad (3.2.15)
$$

$$
+ \int_z^1 U'(\Pi(q, \varepsilon) - D) \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial^2 q} \right) d\varepsilon
$$

$$
- U'(z) \left( \frac{\partial \Pi(q, z)}{\partial q} \right) \frac{dz}{dq} < 0
$$

The the output level that satisfies (3.2.14) is denoted by $q^A$ and the bankruptcy rate responding to $q^A$ is denoted by $z^A$. Therefore, I get

$$
\frac{\partial W}{\partial q} = \int_{z^A}^1 U'(\Pi(q^A, \varepsilon) - D) \left( \frac{\partial \Pi(q^A, \varepsilon)}{\partial q} \right) d\varepsilon = 0 \quad (3.2.14.1)
$$

As Sandmo (1971) points out that the risk averse overweights low demand states and underweights high demand states, one can easily refer that the risk averse manager’s output level $q^A$ must be lower than the risk neutral manager’s $q^N$ if the risk averse’s bankruptcy rate $z^A$ is equal to the risk neutraler’s $z^N$. Only difference from the reasoning of Sandmo (1971) is that the output choice $q$ determines the region between the bankruptcy rate
z and 1. The output levels \( (q^A, q^N) \) at which \( q^A < q^N \) and so \( z^A < z^N \) are consistent with (3.2.12.1) and (3.2.14.1). But the same output levels at which \( q^A = q^N \) and so \( z^A = z^N \), cannot simultaneously satisfy two first order conditions (3.2.12) and (3.2.14) as explained in Sandmo (1971). In particular, one might infer that the output levels \( (q^A, q^N) \) at which \( q^A > q^N \) and so \( z^A > z^N \) are simultaneously consistent with (3.2.12.1) and (3.2.14.1). However, this conjecture is not true, because of the second order condition (3.2.15)\(^1\).

Therefore, a risk averse manager chooses a lower output equilibrium \( q^A (< q^N) \) and a wider interval \([z^A, 1] (\supset [z^N, 1])\) relative to a risk neutraler’s decision, which means that the risk averser’s average demand is worse than one of the risk neutraler.

### 3.2.3 Limited Liability Effect

The total differentiation of first order condition (3.2.14) with respect to \( q \) and \( D \) then yields the comparative static formula of the risk averse managers:

\[
\frac{\partial^2 \Pi}{\partial q^2} = \int_z^1 U'(\Pi(q, \varepsilon) - D) \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right) d\varepsilon + \int_z^1 U'(\Pi(q, \varepsilon) - D) \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial q^2} \right) d\varepsilon - U'(z) \left( \frac{\partial \Pi(q, z)}{\partial q} \right) \frac{dz}{dq} < 0
\]

the last term including the negative sign is positive, which means that increases in output level shortens the range considered (i.e. \([z, 1]\)). On the other hand, the first and second terms are negative, which implies that increases in output level decrease the marginal utility. Thus, the second order condition infers that, when the output level rises, the changes in the bankruptcy rate are smaller relative to changes in the marginal utility. Therefore, the risk averse managers have to choose lower output levels than do the risk neutral managers.

---

\(^1\)In the second order condition,
\[
\frac{dq}{dD} \Bigg|_{\text{Risk Averse}} = - \frac{\partial^2 W}{\partial q \partial D} / \frac{\partial^2 W}{\partial^2 q} > 0 \tag{3.2.16}
\]

The denominator is negative by second-order condition (3.2.15), which means that \(\frac{dq}{dD}\) has the same sign as \(\frac{\partial^2 W}{\partial q \partial D}\). The expression for \(\frac{\partial^2 W}{\partial q \partial D}\) is given by,

\[
\frac{\partial^2 W}{\partial q \partial D} = \int_z^1 \left[ -U''(\Pi(q, \varepsilon) - D) \right] \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right) \, d\varepsilon - U'(z) \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right) \, dz > 0 \tag{3.2.17}
\]

or using (3.2.11.1) and (3.2.11.2),

\[
\frac{dq}{dD} \Bigg|_{\text{R. A.}} = - \frac{\int_z^1 \left[ -U''(\cdot) \right] \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right) \, d\varepsilon - U'(z) \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right)^2 \, dz}{\int_z^1 U''(\cdot) \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right)^2 \, d\varepsilon + \int_z^1 U'(\cdot) \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial q^2} \right) \, d\varepsilon + U'(z) \left( \frac{\partial \Pi(q, \varepsilon)}{\partial q} \right)^2} \tag{3.2.18}
\]

> 0

The intuition associated with this expression is the same as in the case of the risk neutral manager. An increase in debt causes \(z\) to rise, meaning that the range of states over which the firm becomes bankrupt is expanded. With \(U'(\cdot) \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial q \partial \varepsilon} \right) > 0\), it is states with low marginal returns to output that are moved from the region in which equity-holding managers are residual claimants to the bankruptcy region, where debtholders are the residual claimants. Therefore, the managers ignore these low marginal profit states and want output to rise. Thus an increase in debt tends to make equilibrium output rise. This moral hazard which is called ‘limited liability effect’ works whether the managers are risk averse or risk neutral, unless \(U'(\cdot) \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial q \partial \varepsilon} \right)\) is
positive over all states$^2$.

3.3 Effect of Risk Aversion

In this section, I examine how the degree of risk aversion affects the limited liability effect. I compare two comparative statics, which represent the sizes of the debt financing effect on the output strategy. Dividing the denominator and the numerator of (3.2.18) by the marginal utility at the lowest solvency state $U'(z)$, I get,

$$
\frac{dq}{dD}\bigg|_{Risk\ Averse} = - \int_z^1 \left[ \frac{-U''(z)}{U'(z)} \right] \left( \frac{\partial \Pi(q,\varepsilon)}{\partial q} \right) d\varepsilon - \left( \frac{\partial \Pi(q,\varepsilon)}{\partial q} \right) \left( \frac{\partial \Pi(q,\varepsilon)}{\partial \varepsilon} \right) > 0
$$

(3.3.1)

Under the additional assumption where the third derivatives of the utility function ($U'''$) is close to zero, the first term of the numerator in (3.3.1.) is close to zero. Even though the output levels and the bankruptcy rates changes as the degree of the risk aversion, the changes in the last term in the numerator and the last term in the denominator of (3.3.1) are close to zero relative to the changes of the first term and the second term in the denominator. The first and the second terms are central in showing the effect of risk aversion on the debt financing effect.

$^2$This finding implies that the limited liability effect also works in the case of the risk loving managers, because $U'(z) \left( \frac{\partial^2 \Pi(q,\varepsilon)}{\partial q\partial \varepsilon} \right)$ is positive even if the manager is risk loving ($U'' > 0$).
Since the assumption (3.2.6) and the expression (3.2.18), the comparative static formula of the risk neutral manager is given by,

$$\frac{dq}{dD}_{\text{Risk Neutral}} = -\left(\frac{\frac{\partial \Pi(q,z)}{\partial q}}{\frac{\partial \Pi(z)}{\partial z}}\right) \int_{z}^{1} \left(\frac{\partial^2 \Pi(q,z)}{\partial^2 q}\right) d\varepsilon + \left(\frac{\frac{\partial \Pi(z)}{\partial q}}{\frac{\partial \Pi(z)}{\partial \varepsilon}}\right)^2 > 0 \quad (3.3.2)$$

Therefore, the main difference of the limited liability effect between the risk neutralers and the risk aversers depends on the denominators in (3.3.1) and (3.3.2), which induces following relationship:

$$\frac{dq}{dD}_{\text{Risk Neutral}} \geq \frac{dq}{dD}_{\text{Risk Averse}} \Longleftrightarrow -\int_{z}^{1} \left(\frac{\partial^2 \Pi(q,z)}{\partial^2 q}\right) d\varepsilon \leq \int_{z}^{1} \left(-\frac{U''(z)}{U'(z)}\right) \left(\frac{\partial \Pi(q,z)}{\partial q}\right)^2 d\varepsilon - \int_{z}^{1} \left(-\frac{U''(z)}{U'(z)}\right) \left(\frac{\partial^2 \Pi(q,z)}{\partial^2 q}\right) d\varepsilon \quad (3.3.3)$$

Since $U'' < 0$, note that the left hand side (LHS) and the right hand side (RHS) of the inequality (3.3.3) are both positive. The first term of RHS is positive and it rises as the degree of risk aversion rises. However, the second term in RHS of (3.3.3) is always smaller than LHS and it falls as the degree of risk aversion rises, because $U'' < 0$. Then this implies that the inequality of (3.3.3) is mathematically ambiguous.

There are two points which determine the difference of limited liability effect between risk aversers and risk neutralers. The first term of RHS in (3.3.3) implies that, if the risk averter changes the output levels, it affects not only the marginal stock returns but also the marginal utility. This means that, as
debt rises, the risk averse manager increases less output level than does the risk neutraler. However, the comparison between LHS and the second term of RHS in (3.3.3) means that, the risk manager has an incentive to raise output level higher than does the risk neutral manager, because as debt rises, the marginal utility relative to the highest marginal utility at the lowest solvency state $U'(z)$ becomes lower over all solvency states. Therefore, these two confront effects of risk aversion make the results ambiguous.

3.3.1 Simulation

Since the comparison is mathematically ambiguous, through simulation I examine how the risk aversion affects the limited liability effect under restricted conditions: the exponential utility function, the linear demand function with additive uncertainty and zero marginal cost. Assuming the constant absolute risk aversion, I use the exponential function as the utility form:

$$U(x) = B - \exp(-\sigma x + w)$$

where $\sigma > 0$, $B > 0$ and $w > 0$. Therefore, the absolute risk aversion of this function is the constant term, $\sigma$.

$$- \frac{U''(x)}{U'(x)} = \sigma$$

The inverse demand function is given by,

$$P(q, \varepsilon) = a + \varepsilon - bq, \ a > 0, \ b > 0$$
Therefore, the profit function is given by,

$$\Pi(q, \varepsilon) = [a + \varepsilon - bq]q$$

The random shock, $\varepsilon$ is uniformly distributed over $[0, 1]$ as assumed in the previous section. The stock return of the monopoly firm is represented by,

$$\int_z^1 \{(a + \varepsilon - bq)q - D\}d\varepsilon$$

where $z$ is such that $[a + z - bq]q = D$, assuming $0 < z < 1$.

The risk neutraler’s the first order condition is

$$\frac{\partial V}{\partial q} = \int_z^1 (a + \varepsilon - 2bq)d\varepsilon = 0$$

The second order condition of the risk neutral manager is

$$\frac{\partial^2 V}{\partial q^2} = \int_z^1 (-2b)d\varepsilon + \frac{(a + z - 2bq)^2}{q} < 0$$

The expression for $\frac{\partial^2 V}{\partial q \partial D}$ is given by,

$$\frac{\partial^2 V}{\partial q \partial D} = -\frac{(a + z - 2bq)}{q} > 0$$

Therefore, the comparative statics of the risk neutraler’s limited liability effect is as follows:

$$\frac{dq}{dD}_{Risk \ Neutral} = -\frac{-\frac{(a+z-2bq)}{q}}{\int_z^1 (-2b)d\varepsilon + \frac{(a+z-2bq)^2}{q} > 0} \quad (3.3.4)$$

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The risk averter’s first order condition is given by,

\[ \frac{\partial W}{\partial q} = \int_{z}^{1} \sigma e^{-\sigma[(a+\varepsilon-bq)q-D+w]} (a + \varepsilon - 2bq) \, d\varepsilon = 0 \]

The second order condition is

\[ \frac{\partial^2 W}{\partial^2 q} = \int_{z}^{1} \left( -\sigma^2 e^{-\sigma[(a+\varepsilon-bq)q-D+w]} \right) (a + \varepsilon - 2bq)^2 \, d\varepsilon 
+ \int_{z}^{1} \left( \sigma e^{-\sigma[(a+\varepsilon-bq)q-D+w]} \right) (-2b) \, d\varepsilon 
+ \sigma e^{-\sigma[w]}(a + z - 2bq)^2 \frac{q}{q} \]

< 0

The expression for \( \frac{\partial^2 W}{\partial q \partial D} \) is given by,

\[ \frac{\partial^2 W}{\partial q \partial D} = \int_{z}^{1} \left( \sigma^2 e^{-\sigma[(a+\varepsilon-bq)q-D+w]} \right) (a + \varepsilon - 2bq) \, d\varepsilon - \sigma e^{-\sigma[w]}(a + z - 2bq) \frac{q}{q} > 0 \]

Therefore, the comparative statics of the risk averter’s limited liability effect is as follows:

\[ \left. \frac{dq}{dD} (\sigma) \right|_{Risk Averse} = \left[ \begin{array}{c} \int_{z}^{1} \left( \sigma^2 e^{-\sigma[(a+\varepsilon-bq)q-D+w]} \right) (a + \varepsilon - 2bq) \, d\varepsilon \\
-\sigma e^{-\sigma[w]}(a + z - 2bq) \frac{q}{q} \\
\int_{z}^{1} \left( -\sigma^2 e^{-\sigma[(a+\varepsilon-bq)q-D+w]} \right) (a + \varepsilon - 2bq)^2 \, d\varepsilon \\
+ \int_{z}^{1} \left( \sigma e^{-\sigma[(a+\varepsilon-bq)q-D+w]} \right) (-2b) \, d\varepsilon \\
+ \sigma e^{-\sigma[w]}(a + z - 2bq)^2 \frac{q}{q} \end{array} \right] \]

(3.3.5)
Table 3.1: Limited Liability Effect on Degree of Risk Aversion

<table>
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<tr>
<th>$\sigma$</th>
<th>$\frac{dq}{dD}$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>0.13333</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
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<td>-1.98014</td>
<td>-1.98102</td>
</tr>
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<td>-0.01121</td>
<td>-1.81342</td>
<td>-1.82463</td>
</tr>
<tr>
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<td>-0.05976</td>
<td>-1.50896</td>
<td>-1.56872</td>
</tr>
<tr>
<td>0.4</td>
<td>0.15868</td>
<td>-0.10527</td>
<td>-1.38447</td>
<td>-1.48974</td>
</tr>
<tr>
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<tr>
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<td>-1.44930</td>
</tr>
<tr>
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<td>-1.09252</td>
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</tr>
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<td>-0.60844</td>
<td>-1.01635</td>
<td>-1.62479</td>
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<td>-1.02070</td>
<td>-0.94840</td>
<td>-1.96910</td>
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<td>-27.26821</td>
</tr>
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</table>

\[
(1) = \int_z^1 \frac{U''(\zeta)}{U'(\zeta)} \left( \frac{\partial \Pi(q,\varepsilon)}{\partial q} \right)^2 d\varepsilon \\
(2) = \int_z^1 \frac{U''(\zeta)}{U'(\zeta)} \left( \frac{\partial^2 \Pi(q,\varepsilon)}{\partial q^2} \right) d\varepsilon \\
(3) = \int_z^1 \frac{U''(\zeta)}{U'(\zeta)} \left( \frac{\partial \Pi(q,\varepsilon)}{\partial q} \right)^2 d\varepsilon + \int_z^1 \frac{U''(\zeta)}{U'(\zeta)} \left( \frac{\partial^2 \Pi(q,\varepsilon)}{\partial q^2} \right) d\varepsilon
\]
Figure 3.1: Risk Aversion and Limited Liability Effect

For example, suppose that $a = 3.5$, $b = 1$. As $\sigma$ varies, I obtain the values in Table 3.1. As shown in Table 3.1, the risk averser’s limited liability effect is higher than the risk neutraler’s one when $0 < \sigma < 0.8$. However, when $\sigma > 0.9$, the more risk averse the manager is, the lower the limited liability becomes. This fact is graphically shown in Figure 3.1.

Basically the changes of the limited liability depends on the denominator of (3.3.5). As shown in the table, the first term of the risk averser’s denominator, $\int_z^1 \frac{U''(\zeta)}{U'(z)} \left( \frac{\partial \Pi(q,\varepsilon)}{\partial q} \right)^2 d\varepsilon$ increases (in the absolute values) as the risk aversion rises. However, the second term of the risk averser’s denominator, $\int_z^1 \frac{U''(\zeta)}{U'(z)} \left( \frac{\partial^2 \Pi(q,\varepsilon)}{\partial q^2} \right) d\varepsilon$ falls as $\sigma$ rises. If $\sigma \in (0,1)$, then it underweights the first term, $\int_z^1 \frac{U''(\zeta)}{U'(z)} \left( \frac{\partial \Pi(q,\varepsilon)}{\partial q} \right)^2 d\varepsilon$, and it overweight the second
term $\int_{\varepsilon}^{1} \frac{U'(\varepsilon)}{U'(z)} \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial q^2} \right) d\varepsilon$, because $-\frac{U''(\varepsilon)}{U'(z)} \approx \sigma$ but $\frac{U''(\varepsilon)}{U'(z)} \approx 1$ for all $\varepsilon$. On the other hand, if $\sigma > 1$, then it overweights the first term, $\int_{\varepsilon}^{1} \frac{U'(\varepsilon)}{U'(z)} \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial q^2} \right)^2 d\varepsilon$, and it underweights the second term $\int_{\varepsilon}^{1} \frac{U'(\varepsilon)}{U'(z)} \left( \frac{\partial^2 \Pi(q, \varepsilon)}{\partial q^2} \right) d\varepsilon$, because $-\frac{U''(\varepsilon)}{U'(z)} \approx \sigma$ but $\frac{U''(\varepsilon)}{U'(z)} \approx 1$ for all $\varepsilon$. This means that the increases in debt reduces the highest marginal utility, $U'(z)$, and so the risk averser increases the output higher than does the risk neutraler if $0 < \sigma < 1$, because the reducing marginal utility effect of increases in the output is underweighted. If $\sigma > 1$, then the increases in output reduces not only marginal stock returns but also the marginal utility, and so the risk averse increases the output lower than does the risk neutraler, because the increases in debt reduces the highest marginal utility so that the risk averse has an incentive to increases the output more than does the risk neutraler but this effect is underweighted if $\sigma > 1$.

### 3.4 Concluding Remarks

This study examines the effect of risk aversion on the limited liability effect. Using the conventional properties of the risk aversion in the production decision, I have analyzed the behavior of the risk averse managers in the same framework of Brander and Lewis (1986). In this situation, the risk averse managers choose lower output equilibrium than do the risk neutral managers, because the risk averse managers take more weight on the worse states in the production decision. This implies that the conventional theory of the risk aversion is still established in the leveraged firm case.
The comparative statics of the risk averse manager’s decision show that the limited liability effect works even in the risk averse manager case. Although the risk averse concerns the bankrupt more than does the risk neutraler, as the debt rises, the risk averse also focuses on better states so that this manager increases production responding higher demand states. However, the relationship between the degree of risk aversion and the limited liability effect is ambiguous, because the risk managers have the trade-off incentives. Since as debt rises the marginal utility becomes lower over all solvency states relative to the highest marginal utility at the lowest solvency state, the risk manager wants output to rise higher than does the risk neutral manager. This implies that the limited liability effect rises as the degree of risk aversion increases. On the other hand, the risk averse manager has an incentive to increases less output level than does the risk neutraler when debt rises, because the changes in output affect not only the marginal stock returns but also the marginal utility. Therefore, the trade-off relationship between two incentives of the risk averse manager make the results ambiguous. In an example of the constant absolute risk aversion with the linear demand function, if the risk aversion is very low (less than one), the first incentive is greater than the second one so that the risk averse manager want output to rise higher than does the risk neutral manager. The reverse results appear, if the risk aversion is high (greater than one).
Appendices
Appendix A

Chapter 1 Appendix

A.1 Comparative Statics in Repeated Games

The second-order condition of (1.3.12) is given by,

\[
\frac{\partial^2 V_i}{\partial q_{i1}^2} = - \left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial q_{i1}} \right) \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) + \int_{z_{i1}}^{1} \frac{\partial^2 \Pi_{i1}}{\partial q_{i1}^2} \, d\varepsilon_{i1} \tag{A.1.1}
\]

\[
- \beta \left( \frac{\partial^2 z_{i1}}{\partial q_{i1}^2} \right) \left[ (z_{j1}) S_{i2}^M + (1 - z_{j1}) S_{i2}^{BL} \right]
\]

\[
- 2\beta \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) \left[ S_{i2}^M - S_{i2}^{BL} \right]
\]

\[
+ \beta(1 - z_{i1}) \left( \frac{\partial^2 z_{j1}}{\partial q_{i1}^2} \right) \left[ S_{i2}^M - S_{i2}^{BL} \right] < 0
\]

To show the slope of the best response function of the first period output, we need the derivative of the first order condition with respect to the rival firm’s output:

\[
\frac{\partial^2 V_i}{\partial q_{i1} \partial q_{j1}} = - \left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial q_{i1}} \right) \left( \frac{\partial z_{i1}}{\partial q_{j1}} \right) + \int_{z_{i1}}^{1} \frac{\partial^2 \Pi_{i1}}{\partial q_{i1} \partial q_{j1}} \, d\varepsilon_{i1} \tag{A.1.2}
\]

\[
- \beta \left( \frac{\partial^2 z_{i1}}{\partial q_{i1} \partial q_{j1}} \right) \left[ (z_{j1}) S_{i2}^M + (1 - z_{j1}) S_{i2}^{BL} \right]
\]

\[
- \beta \left[ \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) + \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) \right] \left[ S_{i2}^M - S_{i2}^{BL} \right]
\]

\[
+ \beta(1 - z_{i1}) \left( \frac{\partial^2 z_{j1}}{\partial q_{i1} \partial q_{j1}} \right) \left[ S_{i2}^M - S_{i2}^{BL} \right]
\]

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The first two terms on the RHS of (A.1.2) are the same conditions as the BL model. These are both negative; the other terms show that the slope of $q_{i1}$ to $q_{j1}$ is steeper than the BL output function since $\partial^2 z_{j1}/\partial q_{i1} \partial q_{j1}$ is very small relative to the other terms. If demand is linear and marginal cost is constant, equation (A.1.2) is negative.

Similarly, we observe how the reaction function, $q_{i1}$ changes with the leverage $d_i$.

$$
\frac{\partial^2 V_i}{\partial q_{i1} \partial d_i} = - \left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial q_{i1}} \right) \left( \frac{\partial z_{i1}}{\partial d_i} \right) 
- \beta \left( \frac{\partial^2 z_{i1}}{\partial q_{i1} \partial d_i} \right) \left[ (z_{j1})S^{M}_{i2} + (1 - z_{j1})S^{BL}_{i2} \right] 
- \beta \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left( \frac{dS^{M}_{i2}}{dd_i} + (1 - z_{j1}) \frac{dS^{BL}_{i2}}{dd_i} \right) 
- \beta \left( \frac{\partial z_{j1}}{\partial d_i} \right) \left[ S^{M}_{i2} - S^{BL}_{i2} \right] 
+ \beta(1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) \left[ \frac{dS^{M}_{i2}}{dd_i} - \frac{dS^{BL}_{i2}}{dd_i} \right]
$$

(A.1.3)

where

$$
\frac{dS^{M}_{i2}}{dd_i} = \int_0^1 \frac{\Pi^{M}_{i2}(q_{i2}, \varepsilon_{i2}) - d_i d\varepsilon_{i2}}{dd_i} = -1
$$

(A.1.4)

$$
\frac{dS^{BL}_{i2}}{dd_i} = \int_{z_{i2}}^1 \left[ \frac{\partial \Pi_{i2} d_{q_{i2}}}{\partial q_{i2}} \frac{dq_{i2}}{dd_i} + \frac{\partial \Pi_{i2} d_{q_{j2}}}{\partial q_{j2}} \frac{dq_{j2}}{dd_i} - 1 \right] d\varepsilon_{i2}
= \left( \int_{z_{i2}}^1 \frac{\partial \Pi_{i2}}{\partial q_{i2}} d\varepsilon_{i2} \right) \left( \frac{dq_{j2}}{dd_i} \right) - (1 - z_{i2})
$$

(A.1.5)
Plugging (A.1.4) and (A.1.5) into (A.1.3), we get

\[
\frac{\partial^2 V_i}{\partial q_{i1} \partial d_i} = - \left( \frac{\partial \Pi_{i1}(z_{i1})}{\partial q_{i1}} \right) \left( \frac{\partial z_{i1}}{\partial d_i} \right) - \beta \left( \frac{\partial^2 z_{i1}}{\partial q_{i1} \partial d_i} \right) [(z_{j1}) S_{j2}^M + (1 - z_{j1}) S_{j2}^{BL}] + \beta \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) [z_{j1} + (1 - z_{j1})(1 - z_{i2}) - \left( \int_{z_{i2}}^{1} \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\xi_{i2} \right) \left( \frac{dq_{j2}}{dd_i} \right)] - \beta \left( \frac{\partial z_{i1}}{\partial d_i} \right) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) [S_{j2}^M - S_{j2}^{BL}] - \beta (1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) \left[ z_{i2} + \left( \int_{z_{i2}}^{1} \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\xi_{i2} \right) \left( \frac{dq_{j2}}{dd_i} \right) \right]
\]  

(A.1.6)

The effect of firm \( i \)'s debt on \( q_{j1} \) depends on the following derivative,

\[
\frac{\partial^2 V_j}{\partial q_{j1} \partial d_i} = - \beta \left[ \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) \left( \frac{\partial z_{i1}}{\partial d_i} \right) + (1 - z_{j1}) \left( \frac{\partial^2 z_{i1}}{\partial q_{i1} \partial d_i} \right) \right] [S_{j2}^M - S_{j2}^{BL}] - \beta \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) \left[ (z_{i1}) \frac{dS_{j2}^M}{dd_i} + (1 - z_{i1}) \frac{dS_{j2}^{BL}}{dd_i} \right] + \beta (1 - z_{j1}) \left( \frac{\partial z_{i1}}{\partial q_{j1}} \right) \left[ \frac{dS_{j2}^M}{dd_i} - \frac{dS_{j2}^{BL}}{dd_i} \right]
\]  

(A.1.7)

\[= - \beta \left[ \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) \left( \frac{\partial z_{i1}}{\partial d_i} \right) + (1 - z_{j1}) \left( \frac{\partial^2 z_{i1}}{\partial q_{i1} \partial d_i} \right) \right] [S_{j2}^M - S_{j2}^{BL}] - \beta \left[ (1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) + (1 - z_{j1}) \left( \frac{\partial z_{i1}}{\partial q_{j1}} \right) \right] \left( \int_{z_{i2}}^{1} \frac{\partial \Pi_{j2}}{\partial q_{j2}} d\xi_{j2} \right) \left( \frac{dq_{j2}}{dd_i} \right) \]

Signs of (A.1.6) and (A.1.7) are ambiguous, because they depend on \( \beta \), the demand, and cost function forms. Therefore, the debt’s effect on the first-period output levels is ambiguous. When \( \beta = 0 \), \( \partial^2 V_i/\partial q_{i1} \partial d_i > 0 \) and \( \partial^2 V_j/\partial q_{j1} \partial d_i = 0 \). Then, at a low \( \beta \) level (very close to 0), signs of (1.3.27) and (1.3.24) show the same signs as in the BL equilibrium so that \( dq_{i1}/dd_i > 0 \)
and \( dq_{j1}/dd_i < 0 \). It means that when the discount factor is low, two leveraged firms increase output levels not because of predation but because of the limited liability effect.

### A.2 First Order Condition of Optimal Financial Decision

The marginal effect of an increase in \( d_i \) on the present equity value of firm \( i \) is given by

\[
\frac{dV_i}{dd_i} = \frac{dS_{i1}}{dd_i} \tag{A.2.1}
\]

\[
- \beta \left( \frac{\partial z_{i1} dq_{i1}}{\partial q_{i1} dd_i} + \frac{\partial z_{i1} dq_{j1}}{\partial q_{j1} dd_i} + \frac{\partial z_{i1}}{\partial d_i} \right) \left[ (z_{j1})^{SM}_{i2} + (1 - z_{j1})^{SM}_{i2} \right] 
+ \beta (1 - z_{i1}) \left( \frac{\partial z_{j1} dq_{i1}}{\partial q_{i1} dd_i} + \frac{\partial z_{j1} dq_{j1}}{\partial q_{j1} dd_i} \right) \left[ S_{i2}^{M} - S_{i2}^{BL} \right] 
+ \beta (1 - z_{i1}) \left( \frac{dS_{i2}^{M}}{dd_i} + (1 - z_{j1}) \frac{dS_{i2}^{BL}}{dd_i} \right)
\]

where

\[
\frac{dS_{i1}}{dd_i} = - (\Pi_{i1} (q_{i1}, q_{j1}, z_{i1}) - d_i) \left( \frac{\partial z_{i1} dq_{i1}}{\partial q_{i1} dd_i} + \frac{\partial z_{i1} dq_{j1}}{\partial q_{j1} dd_i} + \frac{\partial z_{i1}}{\partial d_i} \right) \tag{A.2.2}
\]

\[
+ \int_{z_{i1}}^{1} \left[ \frac{\partial \Pi_{i1} dq_{i1}}{\partial q_{i1} dd_i} + \frac{\partial \Pi_{i1} dq_{j1}}{\partial q_{j1} dd_i} \right] d\varepsilon_{i1} - \int_{z_{i1}}^{1} 1 d\varepsilon_{i1} 
= \left( \int_{z_{i1}}^{1} \frac{\partial \Pi_{i1}}{\partial q_{i1} d\varepsilon_{i1}} \right) \left[ \frac{dq_{i1}}{dd_i} \right] + \left( \int_{z_{i1}}^{1} \frac{\partial \Pi_{i1}}{\partial q_{i1} d\varepsilon_{i1}} \right) \left[ \frac{dq_{j1}}{dd_i} \right] - (1 - z_{i1}) 
\]

\[
\frac{dS_{i2}^{M}}{dd_i} = \int_{0}^{1} \left[ \frac{\partial \Pi_{i2}^{M} dq_{i2}^{M}}{\partial q_{i2}^{M} dd_i} \right] d\varepsilon_{i2} - \int_{0}^{1} 1 d\varepsilon_{i2} = -1 \tag{A.2.3}
\]
\[
\frac{dS_{i2}^{BL}}{dd_i} = - (\Pi_{i2}(q_{i2}, q_{j2}, z_{i2}) - d_i) \left( \frac{\partial z_{i2}}{\partial q_{i2}} \left[ \frac{dq_{i2}}{dd_i} \right] + \frac{\partial z_{i2}}{\partial q_{j2}} \left[ \frac{dq_{j2}}{dd_i} \right] + \frac{\partial z_{i2}}{\partial d_i} \right) \\
+ \int_{z_{i2}}^{1} \left[ \frac{\partial \Pi_{i2}}{\partial q_{i2}} \frac{dq_{i2}}{dd_i} + \frac{\partial \Pi_{i2}}{\partial q_{j2}} \frac{dq_{j2}}{dd_i} \right] d\varepsilon_{i2} - \int_{z_{i2}}^{1} 1 d\varepsilon_{i2} \\
= \left( \int_{z_{i2}}^{1} \frac{\partial \Pi_{i2}}{\partial q_{i2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{i2}}{dd_i} \right] - [1 - z_{i2}]
\]

Plugging (A.2.2), (A.2.3) and (A.2.4) into (A.2.1), we get

\[
\frac{dV_i}{dd_i} = \left[ \frac{dq_{i1}}{dd_i} \right] \times \left[ \int_{z_{i1}}^{1} \frac{\partial \Pi_{i1}}{\partial q_{i1}} d\varepsilon_{i1} - \beta \left( \frac{dz_{i1}}{dq_{i1}} \right) \left( (z_{j1})S_{i2}^{M} + (1 - z_{j1})S_{i2}^{BL} \right) \\
+ \beta(1 - z_{i1}) \left( \frac{dz_{j1}}{dq_{i1}} \right) \left( S_{i2}^{M} - S_{i2}^{BL} \right) \right] \\
+ \left[ \frac{dq_{j1}}{dd_i} \right] \times \left[ \int_{z_{i1}}^{1} \frac{\partial \Pi_{i1}}{\partial q_{j1}} d\varepsilon_{i1} - \beta \left( \frac{dz_{i1}}{dq_{j1}} \right) \left( (z_{j1})S_{i2}^{M} + (1 - z_{j1})S_{i2}^{BL} \right) \\
- \beta(1 - z_{j1}) \left( \frac{dz_{i1}}{dq_{j1}} \right) \left( S_{i2}^{M} - S_{i2}^{BL} \right) \right] \\
-(1 - z_{i1}) - \beta(1 - z_{i1}) [z_{j1} + (1 - z_{j1})(1 - z_{i2})] \\
- \beta \left( \frac{dz_{i1}}{dd_i} \right) [(z_{j1})S_{i2}^{M} + (1 - z_{j1})S_{i2}^{BL}] \\
+ \beta(1 - z_{i1})(1 - z_{j1}) \left( \int_{z_{i2}}^{1} \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{j2}}{dd_i} \right].
\]
Since the first term in the RHS of (A.2.5) is zero by the first-order condition (1.3.12),

$$\frac{dV_i}{dd_i} = \left[ \frac{dq_{j1}}{dd_i} \right] \times \left[ \int_{z_{i1}}^{1} \frac{\partial \Pi_{i1}}{\partial q_{j1}} d\varepsilon_{i1} - \beta \left( \frac{\partial z_{i1}}{\partial q_{j1}} \right) \left((z_{j1})S^M_{i2} + (1 - z_{j1})S^BL_{i2}\right) \right. $$

$$- \beta (1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) \left(S^M_{i2} - S^BL_{i2}\right) $$

$$- (1 - z_{i1}) - \beta (1 - z_{i1}) \left[z_{j1} + (1 - z_{j1})(1 - z_{i2})\right] $$

$$- \beta \left( \frac{\partial z_{i1}}{\partial d_i} \right) \left[z_{j1}S^M_{i2} + (1 - z_{j1})S^BL_{i2}\right] $$

$$+ \beta (1 - z_{i1})(1 - z_{j1}) \left( \int_{z_{i2}}^{1} \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{j2}}{dd_i} \right] $$

(A.2.6)

The marginal effect of an increase in $d_i$ on the present debt value of firm $i$ is given by

$$\frac{dW_i}{dd_i} = \frac{dD_{i1}}{dd_i} $$

$$- \beta \left( \frac{\partial z_{i1}}{\partial q_{i1}} \frac{dq_{i1}}{dd_i} + \frac{\partial z_{i1}}{\partial q_{j1}} \frac{dq_{j1}}{dd_i} + \frac{\partial z_{i1}}{\partial d_i} \right) \left((z_{j1})D^M_{i2} + (1 - z_{j1})D^BL_{i2}\right) $$

$$+ \beta (1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \frac{dq_{i1}}{dd_i} + \frac{\partial z_{j1}}{\partial q_{j1}} \frac{dq_{j1}}{dd_i} \right) \left(D^M_{i2} - D^BL_{i2}\right) $$

$$+ \beta (1 - z_{i1}) \left( z_{j1} \frac{dD^M_{i2}}{dd_i} + (1 - z_{j1}) \frac{dD^BL_{i2}}{dd_i} \right) $$

(A.2.7)

where

$$D_{i1} = \int_{0}^{z_{i1}} \Pi_{i1}(q_{i1}, q_{j1}, \varepsilon_{i1}) d\varepsilon_{i1} + \int_{z_{i1}}^{1} d_i d\varepsilon_{i1} $$

$$D^M_{i2} = \int_{0}^{1} d_i d\varepsilon_{i2} $$

$$D_{i2} = \int_{0}^{z_{i2}} \Pi_{i2}(q_{i2}, q_{j2}, \varepsilon_{i2}) d\varepsilon_{i2} + \int_{z_{i2}}^{1} d_i d\varepsilon_{i2} $$

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\[
\frac{dD_{i1}}{dd_i} = (\Pi_{i1}(q_{i1}, q_{j1}, z_{i1}) - d_i) \left( \frac{\partial z_{i1}}{\partial q_{i1}} \frac{dq_{i1}}{dd_i} + \frac{\partial z_{i1}}{\partial q_{j1}} \frac{dq_{j1}}{dd_i} + \frac{\partial z_{i1}}{\partial d_i} \right) \\
+ \int_0^{z_{i1}} \left\{ \frac{\partial \Pi_{i1}}{\partial q_{i1}} \frac{dq_{i1}}{dd_i} + \frac{\partial \Pi_{i1}}{\partial q_{j1}} \frac{dq_{j1}}{dd_i} \right\} d\varepsilon_{i1} + \int_{z_{i1}}^{1} 1 d\varepsilon_{i1} \\
= \left( \int_0^{z_{i1}} \frac{\partial \Pi_{i1}}{\partial q_{i1}} d\varepsilon_{i1} \right) \left[ \frac{dq_{i1}}{dd_i} \right] + \left( \int_0^{z_{i1}} \frac{\partial \Pi_{i1}}{\partial q_{j1}} d\varepsilon_{i1} \right) \left[ \frac{dq_{j1}}{dd_i} \right] + (1 - z_{i1})
\]

\[
\frac{dD_{i2}^M}{dd_i} = \int_0^1 (d_i) d\varepsilon_{i2} = 1
\]  (A.2.9)

\[
\frac{dD_{i2}^{BL}}{dd_i} = (\Pi_{i2}(q_{i2}, q_{j2}, z_{i2}) - d_i) \left( \frac{\partial z_{i2}}{\partial q_{i2}} \frac{dq_{i2}}{dd_i} + \frac{\partial z_{i2}}{\partial q_{j2}} \frac{dq_{j2}}{dd_i} + \frac{\partial z_{i2}}{\partial d_i} \right) \\
+ \int_0^{z_{i2}} \left\{ \frac{\partial \Pi_{i2}}{\partial q_{i2}} \frac{dq_{i2}}{dd_i} + \frac{\partial \Pi_{i2}}{\partial q_{j2}} \frac{dq_{j2}}{dd_i} \right\} d\varepsilon_{i2} + \int_{z_{i2}}^{1} 1 d\varepsilon_{i2} \\
= \left( \int_0^{z_{i2}} \frac{\partial \Pi_{i2}}{\partial q_{i2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{i2}}{dd_i} \right] + \left( \int_0^{z_{i2}} \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{j2}}{dd_i} \right] + (1 - z_{i2})
\]  (A.2.10)

Plugging (A.2.8), (A.2.9) and (A.2.10) into (A.2.7), we get

\[
\frac{dW_i}{dd_i} = \left[ \frac{dq_{i1}}{dd_i} \right] \times \left[ \int_0^{z_{i1}} \frac{\partial \Pi_{i1}}{\partial q_{i1}} d\varepsilon_{i1} - \beta \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left( (z_{j1}) D_{i2}^M + (1 - z_{j1}) D_{i2}^{BL} \right) \right] \\
+ \beta(1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) \left( D_{i2}^M - D_{i2}^{BL} \right) \\
+ \left[ \frac{dq_{j1}}{dd_i} \right] \times \left[ \int_0^{z_{i1}} \frac{\partial \Pi_{i1}}{\partial q_{j1}} d\varepsilon_{i1} - \beta \left( \frac{\partial z_{i1}}{\partial q_{j1}} \right) \left( (z_{j1}) D_{i2}^M + (1 - z_{j1}) D_{i2}^{BL} \right) \right] \\
+ \beta(1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{j1}} \right) \left( D_{i2}^M - D_{i2}^{BL} \right) \\
+ (1 - z_{i1}) + \beta(1 - z_{i1}) \left[ (z_{j1}) + (1 - z_{j1})(1 - z_{i2}) \right] \\
- \beta \left( \frac{\partial z_{i1}}{\partial d_i} \right) \left[ (z_{j1}) D_{i2}^M + (1 - z_{j1}) D_{i2}^{BL} \right] \\
+ \beta(1 - z_{i1}) (1 - z_{i2}) \left\{ \left( \int_0^{z_{i2}} \frac{\partial \Pi_{i2}}{\partial q_{i2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{i2}}{dd_i} \right] \right\} \\
+ \left( \int_0^{z_{i2}} \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{j2}}{dd_i} \right] \}
\]
By (A.2.6) and (A.2.11), the marginal effect of an increase in \( d_i \) on the whole present value of firm \( i \), \( dY_i/\dd_i \) is given by

\[
\frac{dY_i}{\dd_i} = \left[ \frac{dq_{i1}}{\dd_i} \right] \left[ \int_0^{z_{i1}} \frac{\partial \Pi_{i1}}{\partial q_{i1}} d\varepsilon_{i1} \right]
\]

\[
- \beta \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left\{ (z_{j1})d_i + (1 - z_{j1}) \int_0^{z_{i2}} \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} + (1 - z_{j1})(1 - z_{i2}) d_i \right\}
\]

\[
- \beta (1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) \left\{ (z_{i2})d_i - \int_0^{z_{i2}} \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right\}
\]

\[
+ \left[ \frac{dq_{i1}}{\dd_i} \right] \left[ \int_0^1 \frac{\partial \Pi_{i1}}{\partial q_{i1}} d\varepsilon_{i1} \right]
\]

\[
- \beta \left( \frac{\partial z_{i1}}{\partial q_{i1}} \right) \left\{ (z_{j1}) \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} + (1 - z_{j1}) \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right\}
\]

\[
- \beta (1 - z_{i1}) \left( \frac{\partial z_{j1}}{\partial q_{i1}} \right) \left\{ \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} - \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right\}
\]

\[
- \beta \left( \frac{\partial z_{i1}}{\partial d_i} \right) \left[ (z_{j1}) \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} + (1 - z_{j1}) \int_0^1 \Pi_{i2}(\varepsilon_{i2}) d\varepsilon_{i2} \right]
\]

\[
+ \beta (1 - z_{i1})(1 - z_{j1}) \left[ \left( \int_0^{z_{i2}} \frac{\partial \Pi_{i2}}{\partial q_{i2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{i2}}{\dd_i} \right] + \left( \int_0^1 \frac{\partial \Pi_{i2}}{\partial q_{j2}} d\varepsilon_{i2} \right) \left[ \frac{dq_{j2}}{\dd_i} \right] \right]
\]
Appendix B

Chapter 2 Appendix

B.1 Proof of Lemma 2.3.4

Since both firms choose the same debt level, each firm’s bankruptcy rate depends on its output level. If a firm $i$ chooses the output level $q_i$ such that $d_i/q_i < d_j/q_j$, then its own bankruptcy rate, $z_i$ is lower than the rival’s, $z_j$ ($z_i < z_j$). If the firm $i$ chooses the output level, $q_i = \left( \frac{d_i}{d_j} \right) q_j$, then its own bankruptcy rate, $z_i = z_j$. Suppose that there is the output equilibrium $(q_i^0, q_j^0)$ which satisfies $d_i/q_i = d_j/q_j$ under different debt levels $(d_i, d_j)$. Denote that $z^0 \equiv z_i(q_i^0, q_j^0; d_i, d_j) = z_j(q_i^0, q_j^0; d_i, d_j)$. Since both firms choose $(q_i^0, q_j^0)$, then the firm $i$’s whole stock value is as follows; $V_i = S_i + \beta (1 - z_i) \Pi^C$ and the first order condition is,

$$\frac{dV_i}{dq_i} = \frac{dS_i}{dq_i} - \beta \left( \frac{dz_i}{dq_i} \right) \Pi^C = 0$$

Denote that $\tilde{q}_i \equiv q_i^0 + \eta$ where $\eta$ is very tiny positive term. If both firms keep the pure strategy output level $(q_i^0, q_j^0)$,

$$V_i(q_i^0, q_j^0; d_i, d_j) = S_{ii}(q_i^0, q_j^0; d_i, d_j) + \beta (1 - z^0) \Pi^C$$

If the firm $i$ takes $\tilde{q}_i$ given the firm j’s output level $q_j^0$,

$$V_i(q_i^0 + \eta, q_j^0; d_i, d_j) = S_{ii}(q_i^0 + \eta, q_j^0; d_i, d_j) + \beta [(z_j - z_i) \Pi^M + (1 - z_j) \Pi^C]$$

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Note that the payoff function is continuous, if firm $i$ changes from $q_i^0$ to $\tilde{q}_i$.

Then the difference between two whole stock values is,

$$V_i(q_i^0 + \eta, q_j^0; d_i, d_j) - V_i(q_i^0, q_j^0; d_i, d_j)$$

$$= S_{\Pi}(q_i^0 + \eta, q_j^0; d_i, d_j) - S_{\Pi}(q_i^0, q_j^0; d_i, d_j) + \beta \left[(z_j - z_i) \Pi^M - (z^0 - z_j) \Pi^C\right]$$

$$= S_{\Pi}(q_i^0 + \eta, q_j^0; d_i, d_j) - S_{\Pi}(q_i^0, q_j^0; d_i, d_j) + \beta \left[(z_j - z^0 - z_i + z^0) \Pi^M - (z^0 - z_j) \Pi^C\right]$$

Then,

$$\lim_{\eta \to 0^+} \frac{V_i(q_i^0 + \eta, q_j^0; d_i, d_j) - V_i(q_i^0, q_j^0; d_i, d_j)}{\eta}$$

$$= \lim_{\eta \to 0^+} \frac{S_{\Pi}(q_i^0 + \eta, q_j^0; d_i, d_j) - S_{\Pi}(q_i^0, q_j^0; d_i, d_j)}{\eta}$$

$$+ \lim_{\eta \to 0^+} \frac{\beta \left[(z_j - z^0 - z_i + z^0) \Pi^M - (z^0 - z_j) \Pi^C\right]}{\eta}$$

$$> 0$$

The first term is the positive, because of $\frac{dS_{\Pi}}{dq_i dq_j} < 0$ and $\frac{dS_{\Pi}}{dq_i} < 0$. Since $\frac{dS_{\Pi}(q^BL, q^BL)}{dq_i} = 0$, the derivative $\frac{dS_{\Pi}}{dq_i}$ around $(q_i^0, q_j^0)$ is positive, if $q_i^0 < q_i^{BL}$.

Since $\eta$ is a very tiny positive term, the second part of above equation is positive as well. Therefore, each firm can increase its own stock value through raising its output level. Hence no one keeps this pure symmetric output level.

If $q_i^0 = q_i^{BL}$, the Brander and Lewis equilibrium given $d_i$ and $d_j$, the first term of the FOC must be zero but the second term is negative. Then $(q_i^{BL}, q_j^{BL})$ could not be the output equilibrium.

If $q_i^0 > q_i^{BL}$, the first term of FOC is negative as well as the second term is, because of the second order condition and the other condition, $\frac{d^2 S_{\Pi}}{dq_i dq_j} < 0$. 95
Therefore any symmetric output level which are greater than the Brander and Lewis equilibrium could not be the output equilibrium either.

If \( q_i^0 < q_i^{HL} \), the second order condition might be satisfied but this output level is not the best response for each firm as shown above.

Suppose \((q_i^H, q_j^L)\) where \( d_i / q_i^H < d_j / q_j^L \). We can show that there exists \((q_i^H, q_j^L)\) which satisfies two firms’ first order conditions. The first order condition for the firm \( i \) is,

\[
\frac{dV_i}{dq_i}(q_i^H, q_j^L | d_i, d_j) = \frac{dS_{i1}}{dq_i} + \beta \left[ \left( \frac{dz_j}{dq_i} - \frac{dz_i}{dq_i} \right) \Pi^M - \frac{dz_j}{dq_i} \Pi^C \right] = 0
\]

\[
\Leftrightarrow \frac{dS_{i1}}{dq_i} + \beta \left[ \frac{d}{q_i^2} \Pi^M - b \Pi^C \right] = 0
\]

and the FOC for \( j \) is,

\[
\frac{dV_j}{dq_j}(q_i^H, q_j^L | d_i, d_j) = \frac{dS_{j1}}{dq_j} - \beta \frac{dz_j}{dq_j} \Pi^C = 0
\]

\[
\Leftrightarrow \frac{dS_{j1}}{dq_j} - \beta \left( -\frac{d}{q_j^2} + b \right) \Pi^C = 0
\]

The SOC is,

\[
\frac{d^2 V_i}{d^2 q_i}(q_i^H, q_j^L | d_i, d_j) = \frac{d^2 S_{i1}}{d^2 q_i} + \beta \left[ \frac{d^2 z_j}{d^2 q_i} (\Pi^M - \Pi^C) - \frac{d^2 z_i}{d^2 q_i} \Pi^M \right]
\]

\[
= \frac{d^2 S_{i1}}{d^2 q_i} - \beta \left( \frac{d}{q_i^3} \right) \Pi^M < 0
\]

\[
\frac{d^2 V_j}{d^2 q_j}(q_i^H, q_j^L | d_i, d_j) = \frac{d^2 S_{j1}}{d^2 q_j} - \beta \frac{d^2 z_j}{d^2 q_j} \Pi^C = \frac{d^2 S_{j1}}{d^2 q_j} - \beta \left( \frac{d}{q_j^3} \right) \Pi^C
\]

\[< 0\]
The cross derivatives are,
\[
\frac{d^2V_i}{dq_i dq_j} (q_i^H, q_j^L | d_i, d_j) = \frac{d^2 S_{i1}}{dq_i dq_j} + \beta \left[ \frac{d^2 z_j}{dq_i dq_j} (\Pi^M - \Pi^C) - \frac{d^2 z_i}{dq_i dq_j} \Pi^M \right]
\]
\[
= \frac{d^2 S_{i1}}{dq_i dq_j} < 0
\]
\[
\frac{d^2V_j}{dq_j dq_i} (q_i^H, q_j^L | d_i, d_j) = \frac{d^2 S_{j1}}{dq_j dq_i} - \beta \frac{d^2 z_j}{dq_j dq_i} \Pi^C = \frac{d^2 S_{j1}}{dq_j dq_i} < 0
\]

Then,
\[
\frac{d^2V_i}{d^2q_i} \frac{d^2V_j}{d^2q_j} - \frac{d^2V_i}{dq_i dq_j} \frac{d^2V_j}{dq_j dq_i} > 0
\]

Therefore, the output pair, \((q_i^H, q_j^L)\) satisfying two FOCS is stable and one equilibrium given the debt levels, \((d_i, d_j)\).

Similarly, we can show the existence of the other equilibrium \((q_i^L, q_j^H)\)

### B.2 LLE given \(d^*\) and \(\beta\)

If firm \(j\) raises the debt level and firm \(i\) keeps \(d^*\), the FOCS are,

\[
\frac{dV_i}{dq_i} = \frac{dS_{i1}}{dq_i} + \beta \left[ \left( \frac{dz_j}{dq_i} - \frac{dz_i}{dq_i} \right) \Pi^M - \frac{dz_j}{dq_i} \Pi^C \right] = 0 \tag{B.2.1}
\]
\[
\frac{dV_j}{dq_j} = \frac{dS_{j1}}{dq_j} - \beta \frac{dz_j}{dq_j} \Pi^C = 0
\]

and the SOCs are,

\[
\frac{d^2V_i}{d^2q_i} = \frac{d^2 S_{i1}}{d^2q_i} + \beta \left[ \frac{d^2 z_i (\Pi^M - \Pi^C)}{d^2q_i} - \frac{d^2 z_i \Pi^M}{d^2q_i} \right] < 0 \tag{B.2.2}
\]
\[
\frac{d^2V_j}{d^2q_j} = \frac{d^2 S_{j1}}{d^2q_j} - \beta \frac{d^2 z_j \Pi^C}{d^2q_j} < 0
\]

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The cross derivatives are,
\[
\frac{d^2 V_i}{dq_i dq_j} = \frac{d^2 S_{i1}}{dq_i dq_j} + \beta \left[ \frac{d^2 z_j}{dq_i dq_j} (\Pi^M - \Pi^C) - \frac{d^2 z_i}{dq_i dq_j} \Pi^M \right] = \frac{d^2 S_{i1}}{dq_i dq_j} < 0 \quad \text{(B.2.3)}
\]
\[
\frac{d^2 V_j}{dq_i dq_j} = \frac{d^2 S_{j1}}{dq_i dq_j} - \beta \frac{d^2 z_j}{dq_i dq_j} \Pi^C = \frac{d^2 S_{j1}}{dq_j dq_i} < 0
\]

Then,
\[
\frac{d^2 V_i d^2 V_j}{d^2 q_i d^2 q_j} - \frac{d^2 V_i}{dq_i dq_j} \frac{d^2 V_j}{dq_j dq_i} > 0 \quad \text{(B.2.4)}
\]

From the FOCs of \((q_i^{H}(d^*, d^*), q_i^{L}(d^*, d^*))),
\[
\frac{d^2 V_i}{dq_i dd_j} = 0 \quad \text{(B.2.5)}
\]
\[
\frac{d^2 V_j}{dq_j dd_j} = \frac{d^2 S_{j1}}{dq_j dd_j} - \beta \left( \frac{d^2 z_j}{dq_j dd_j} \right) \Pi^C > 0
\]

Then,
\[
\frac{dq_i^{H}}{dd_j} = \left( \frac{d^2 V_j}{dq_j dd_j} \right) \left( \frac{d^2 V_i}{dq_i dq_j} \right) / G < 0 \quad \text{(B.2.6)}
\]
\[
\frac{dq_j^{L}}{dd_j} = - \left( \frac{d^2 V_j}{dq_j dd_j} \right) \left( \frac{d^2 V_i}{d^2 q_i} \right) / G > 0
\]

where, \(G = \frac{d^2 V_i d^2 V_j}{d^2 q_i d^2 q_j} - \frac{d^2 V_i}{dq_i dq_j} \frac{d^2 V_j}{dq_j dq_i} \)

\[
\left[ \frac{dV_j}{dd_j} \right]_{d^*} = \left[ \frac{dq_j^{L}}{dd_j} \right] \left\{ \int_{z_j}^{1} \frac{\partial \Pi_j}{\partial q_j} d\epsilon - \beta \left( \frac{\partial z_j}{\partial q_j} \right) \Pi^C \right\} + \left[ \frac{dq_j^{H}}{dd_j} \right] \left\{ \int_{z_j}^{1} \frac{\partial \Pi_j}{\partial q_i} d\epsilon - \beta \left( \frac{\partial z_j}{\partial q_i} \right) \Pi^C \right\} - (1 - z_j) - \beta \left( \frac{\partial z_j}{\partial d_j} \right) \Pi^C \quad \text{(B.2.7)}
\]
\[
\begin{align*}
\left[ \frac{dW_j}{dd_j} \right]_{d^*} &= \left[ \frac{dq_i^L}{dd_j} \right] \left\{ \int_{0}^{z_j} \frac{\partial \Pi_j}{\partial q_j} \, dz \right\} + \left[ \frac{dq_i^H}{dd_j} \right] \left\{ \int_{z_j}^{1} \frac{\partial \Pi_j}{\partial q_i} \, dz \right\} + (1 - z_j) \quad (B.2.8) \\
\left[ \frac{dY_j(q_i^H, q_i^L)}{dd_j} \right]_{(d^*, d^*)} &= \left[ \frac{dq_i^L}{dd_j} \right] \left\{ \int_{0}^{z_j} \frac{\partial \Pi_j}{\partial q_i} \, dz \right\} + \left[ \frac{dq_i^H}{dd_j} \right] \left\{ \int_{0}^{1} \frac{\partial \Pi_j}{\partial q_i} \, dz \right\} \quad (B.2.9)
\end{align*}
\]
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