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Managing Vertical and Horizontal Supply Chain Relationships in the Absence of Formal Contracts

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Managing Vertical and Horizontal Supply Chain Relationships in the Absence of Formal Contracts

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Managing Vertical and Horizontal Supply Chain Relationships in the Absence of Formal Contracts

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The dissertation consists of three essays that explore signaling mechanisms for coordinating inter and intra supply chain relationships when formal contracts either are not used, or cannot be used. In the first essay, we examine the role of performance based compensation in encouraging strategic investment from suppliers. By altering the senior management's incentives, a compensation plan can serve as a credible commitment to higher levels of output from the firm. Such a commitment can motivate suppliers to enter the industry and invest in capacity. The analysis shows

that, by providing managers with appropriately designed compensation, a firm's shareholders can increase the return on their investment.

In the second essay, we consider the technology licensing in the context of complementary interactions. We investigate how the firm with patent protection can benefit from licensing its technology to other manufacturers even if these other firms do not enjoy a cost advantage. Licensing can provide a credible commitment to the availability of the patent protected product, thereby encouraging output of the complement. We show the conditions under which a firm can earn more from licensing its technology to firms than it can by serving the market as a monopolist. In addition, we explore alternative types of license arrangements in the study.

In the last essay, we investigate the product line strategy under complementarity. If the product is differentiable, a monopolist under strong enough complementary effects would provide a broader product line than he would if the demand for his product was independent of other markets. In addition, we show that providing a broader product line and technology licensing are strategic complements to each other. Finally, the quality decision is studied when the firm licenses the technology to other manufacturers.

Contents

Chapter 1

Introduction

Every company operates in a bio-environment where it interacts directly or indirectly with a number of business entities. Such interactions could be vertical and direct, for instance, a buyer-supplier relationship or a joint promotion by a retailer and a manufacturer. All these fall into the field of Supply Chain Management and have been studied intensively by scholars in Operations Management, Marketing, and Economics. There are also plenty of examples of indirect interactions where one firm's strategic move increases or decreases another firm's welfare although these two firms have no direct business relationship involving payment obligations or transferring of a good. Such indirect, usually horizontal, connections often take form of complementary market interactions. Although we have developed profound knowledge on complementary effects from a social welfare perspective, the strategic considerations of a firm involved in a complementary relationship have not yet been thoroughly studied and that is one of the focuses of this dissertation.

It is not uncommon that formal contracts are absent in direct or indirect interactions. The major obstacles to formal contracting are usually issues associated with enforceability and verifiability, which have been discussed in detail in Chapter 2.

When interactions are indirect, it is even more difficult for firms to form a contractual relationship. For example, as firms involved in indirect complementary effects do not exchange goods or services at all, they may not be able to enter formal contracts with each other. This work examines several coordination mechanisms when formal contracts are not used or cannot be used.

In many industries, a significant portion of senior management's compensation is based on the performance of the firm. Such compensation is commonly explained as a means of providing managers with incentives to exert effort to increase the profits of the firm. However, in Chapter 2, we explore an alternative role that such performance based compensation can play: By altering the senior management's incentives, a compensation plan can serve as a credible commitment to higher levels of output from the firm. Such a commitment may be strategically important as a means of encouraging suppliers to enter the industry and invest in capacity, particularly in industries where firms are reluctant to engage in long term supply contracts. We examine the consequences of insufficient entry of suppliers and show that, by providing managers with appropriately designed compensation, a firm's owners can increase the return on their investment.

In Chapter 3, we consider a setting in which a firm has patent protection for a given product technology that would permit it to produce the product as a monopolist manufacturer. The product is durable, e.g. a high definition television, but interacts with complements, e.g. high definition programming, that are produced by other firms. We investigate how the firm with patent protection can benefit from licensing its technology to other manufacturers even if these other firms do not enjoy a cost advantage. To the extent that the licensing agreement is public information, it can provide a credible commitment to the availability of the patent protected product, thereby encouraging output of the complement. We show the conditions under which a firm can earn more from licensing its technology to firms than it can by serving the market as a monopolist. In addition, we explore alternative types of license arrangements in the study.

In Chapter 4, we extend our results in Chapter 3 to a situation where the durable goods manufacturer has the opportunities to introduce a low end version of the product to the market. Our results show that when the complementary effects are strong, the durable goods manufacturer under complementarity may provide a broader product line than a manufacturer whose product is independent of other markets. A broader product line serves as a credible commitment to a higher future total sales. In addition, we establish that providing a broader product line and licensing arrangement, both of which can convey information about higher future output to the complementary product producer, are strategic complements to each other. Finally, we study the firm's quality decision in the context of technology licensing under complementary effects.

Chapter 2

The Effect of Managerial Incentives on Supply Chain Capacity and Performance

2.1 Introduction

The arguments for linking managerial compensation to the financial performance of the firm are typically based on creating incentives for managers to act in ways that are consistent with maximizing shareholder value. While these arguments are certainly valid, we identify and explore an additional role that such performance based compensation can play as a mechanism for the owners (i.e. shareholders) of a firm to encourage higher levels of investment from suppliers. We show that, for firms that either cannot or will not engage in long term contracts with their suppliers, the use of performance based incentive plans can provide a mechanism for mitigating the hold-up problem.

It is well known that when a firm's suppliers have insufficient capacity, it can suffer from lost sales and poor financial performance. For example, in 2003, a shortage of capacity for flash memory chips caused half of the manufacturers of flash memory devices and MP3 players to stop production.[1] In 2004, Qualcomm, the world's largest chip manufacturer without its own manufacturing plant, was unable to meet all of its customers' demands for its patented CDMA technology due to a shortage of foundry capacity at its suppliers. [2]

For a foundry, it can cost more than $2 billion to set up a production line[3] and the equipment is highly specific. Once a foundry pays these irreversible setup costs, the fabless semiconductor companies are well positioned to negotiate a price very close to the foundry's average variable cost, especially if demand turns out to be weak. Moreover, many of these fabless semiconductor firms are loath to enter contracts that are of a sufficiently long term to guarantee that suppliers will cover their fixed investment costs.

In order to distinguish between long and medium term contracts, we assume that information about demand increases over time and that capacity decisions must precede production decisions which must precede the realization of demand. Within this context, we define a *long-term* contract as one in which the buyer commits to a quantity and a price prior to a supplier's investment in capacity, possibly allowing for contingencies if they can be specified and verified. We define a *medium-term* contract as one in which the buyer commits to a quantity and price after the supplier's investment but prior to the realization of demand. By these definitions, a medium-term contract addresses the supplier's inventory risk by protecting him against producing items that will go unsold, while a long-term contract addresses the supplier's capacity risk by protecting him against making an investment in capacity that cannot be recovered. In practice, there are many

[1] "Samsung NAND Chips Going out of Stock", *21rst Century Business Herald*, page 19, August 29, 2003.

[2] "US Chip Group Signals Shortage." *Financial Time*s. April 23, 2004.

[3] "US Chip Group Signals Shortage." *Financial Time*s. April 23, 2004.

firms that avoid long-term contracts with their suppliers. For example, in a recent discussion with a fabless semi-conductor manufacturer, Sigma Tel, we learned that they avoid providing suppliers with any sort of assurances about quantities that they will buy. This may be due, at least in part, to the difficulty of specifying and verifying various contingencies. However, rather than questioning whether and how long-term contracts might be used, our purpose is to investigate one way in which a firm might encourage its suppliers to invest in capacity without engaging in long-term contracts with them.

We propose that, in the absence of long-term contracts, one potential role of performance based compensation is to alter the incentives of management in order to encourage suppliers to invest in capacity. Our focus is on threshold-based incentive compensation schemes, in which at least some components of compensation for the manager require that performance exceed a certain threshold. As discussed in Holthausen et al. (1995), such schemes are common in practice. One reason for our interest in threshold-based performance compensation schemes is their obvious parallel to corporate stock options. In recent years, stock options have been included in the compensation of large numbers of upper and middle managers. The International Employee Stock Options Coalition estimates that 14 million Americans hold employee stock options (http://www.savestockoptions.org). Stock options reward the manager only when she is able to increase the value of the firm above the exercise price of the options. To the extent that the value of the firm is based on its current financial performance, the issuance of stock options would be one means of implementing a threshold based incentive compensation scheme. Moreover, there is increasing pressure being placed on publicly traded firms to expense employee stock options, which helps to insure that an employee incentive program is observable.

To investigate the role that an incentive based compensation plan can play in encouraging suppliers to invest in capacity, we adopt a simplistic model of the manager, in which her only role is to observe some preliminary information about demand, and determine a quantity to procure from the firm's suppliers. Because this preliminary demand information is observable only to the manager, the owners of the firm cannot make the quantity decision themselves. Nor is it possible for either the owners or the manager to enter contracts with suppliers that are contingent upon this demand signal. One of the questions that we address is that of whether a *threshold* incentive plan, which specifies a minimum level of performance below which the manager receives nothing, can be preferable to one that provides a reward that is linear in the performance of the firm. This is similar to the issue of whether it is better to provide a manager with stock options than to simply grant immediate ownership of shares of the firm's stock. Our purpose is not to suggest that incentive compensation is preferable to long term contracts or other mechanisms for encouraging suppliers to invest in capacity. Rather, we seek only to highlight the fact that *one* of the roles of incentive compensation can be to encourage supplier investment.

The remainder of our paper is organized as follows: After reviewing the related literature, we develop a model of the interaction between a firm and its suppliers. We then show how, in the absence of an effective managerial incentive plan or long term contract, fewer than the efficient number of suppliers will invest and enter to serve the buyer. In Section 2.5.1, we demonstrate how performance based compensation can alter the incentives for senior management and mitigate this hold-up problem. Finally, we provide a discussion of our results in Section 2.6 .

2.2 Related Literature

A number of remedies have been proposed for the *hold-up problem* which can exist when a firm has an opportunity to invest in an asset that is specific to its relationship with a given trading partner. Once the investment is made, the firm will be in a weak bargaining position with respect to its partner. Therefore, in the absence of any credible guarantee from its trading partner to refrain from opportunistic behavior, the firm will tend to under invest, relative to what would be optimal for the combined welfare of the two firms. The most common remedies that have been proposed for the hold-up problem are vertical integration and long term contracts.

Vertical integration is perhaps the most obvious remedy for the holdup problem, e.g. Kleindorfer & Knieps (1982), Williamson (1975). When the parties to trade are jointly owned, they share a common objective function so that neither party has an incentive to act in a manner that benefits itself at the expense of the other, eliminating the potential for hold-up. However, dis-economies of scope and/or dis-economies of scale often make vertical integration impractical.

Long term contracts can be a remedy to the hold-up problem to the extent that future contingencies can be fully specified. When these contingencies can be specified, if the parties to trade can agree on how each one should behave under each contingency and to a mechanism for enforcing this agreement, then the rights of the holders of the specific assets can be protected. See, for example, Alchian & Woodward (1988) and Klein et al. (1978). Unfortunately, as discussed in Milgrom & Roberts (1992), it is often difficult to adequately anticipate all future contingencies, and even when the major contingencies can be specified, it can be difficult to identify measures that are verifiable or contractible.

Several authors have considered how specific contractual forms between buyers and sellers in a supply chain affect their incentives to invest in either demand enhancement, cost reduction, or capacity. Taylor (2002) examines how a manufacturer can offer a threshold-based channel rebate to a newsvending retailer to provide appropriate incentives to affect both his ordering decision and his exertion of effort. Gilbert & Cvsa (2003) discuss how to use strategic commitment to price to stimulate downstream innovation in a supply chain. Plambeck & Taylor (2005) address the issue of stimulating downstream investment in a supply chain, but they address the trade-off between eliminating the hold-up problem by vertically integrating versus pooling manufacturing capacity by allowing a single contract supplier to sell to multiple downstream OEMS. Two other papers consider the implications of different forms of penalties, i.e. breach remedies, when one of the parties to trade cannot deliver what it has promised in the contract. Plambeck & Taylor (October 2004) is one of them. This paper studies how to design appropriate supply contracts under different breach remedies so that the buyer and the suppliers would make first best investments in R&D and capacity respectively. In the other paper, Lyon & Huang (2002) study the impact of renegotiation and breach remedies in situations in which both the buyer and the seller must make an investment.

An important component of our model is a compensation scheme that rewards a newsvending manager only if profit exceeds some specified level. A number of researchers, e.g. Geoffrion (1967), Lau (1980), Lau & Lau (1988), Li et al. (1990), Li et al. (1991), have examined the effects of similar threshold-based objectives in newsvendor inventory contexts. Agrawal & Tsay (2001) provide a nice review of this work and they themselves consider an issue closely related to ours. They consider a compensation scheme that rewards a newsvending manager only if the profit exceeds a threshold. Like us, they consider how this compensation scheme affects the manager's interactions with suppliers. However, they focus on the extent to which the manager will be motivated to negotiate for a lower wholesale price and do not consider how the scheme would affect a supplier's willingness to invest in capacity.

Our work is closest to the literature on what actions a firm can take to encourage investment from a competitive supply industry when long term contracts cannot be used, presumably because of enforcement or verifiability issues. Von Ungern-Sternberg (1988) shows that it can be beneficial for a monopolist to over-invest in its own capacity, relative to the first-best level of investment, as a way to credibly commit to a higher level of output and encourage more suppliers to enter. Subramaniam (1998) shows that debt financing can provide a monopolist with an alternative mechanism for committing to a higher level of output and encouraging supplier entry.

Our model of the supply industry is similar to those of Von Ungern-Sternberg (1988) and Subramaniam (1998), but we do not take it for granted that the management of the firm necessarily seeks to maximize shareholder profits. Instead we focus on the development of a compensation plan that provides appropriate incentives for management and also mitigates the hold-up problem with respect to suppliers. Thus, we specifically recognize the interaction among the interests of owners, managers, and suppliers as well as the fact that those interests may be distinct.

2.3 The Basic Model

In order to examine the question as to the extent to which a manager's compensation scheme can be used to encourage suppliers to invest in capacity, we consider a situation in which a firm (the buyer) faces a single period of uncertain demand.

The retail price of the end good is p and the buyer's cost of processing each unit of the final product is c. In addition to its own processing cost, the buyer depends upon a competitive supply industry. Without loss of generality, we assume that one unit of input from this supply industry is required for each unit of output from the buyer. To enter the supply industry, a supplier must make an irreversible investment of $\$K$. After entry, each supplier faces a convex marginal cost of production, $s(q)$, where q is its volume of output, and $s(q)$ is increasing in the relevant range of output.

We further assume that the lead time for entering and building capacity is longer than the production lead time. Therefore, some additional information about demand may become available between the time when suppliers enter and the time at which a commitment must be made to a production quantity. In reality, such information may be based on a variety of sources, including: general economic conditions, market responses to other related products, direct communications with customers, etc. Access to this sort of information typically requires a substantial level of involvement in the details of running the business, and hence may not be easily observed, let alone verified, by the owners of the buying firm or the suppliers. In our model, the manager's primary role is to observe this information and respond to it by determining a quantity of output. Although we acknowledge that this is a simplification of reality, it is useful for demonstrating how the owners of the buying firm can design a compensation scheme that serves their own interests by encouraging suppliers to invest in capacity.

We assume that the sequence of events is as follows: Prior to the entry of suppliers, the owners of the buyer determine a compensation scheme for the manager. (In practice, there may be many managers, but for simplicity, we use the singular noun.) Note that, if the compensation plan involves the immediate issuance of corporate stock or stock options, it may be difficult, if not impossible, for the owners to renege. In response to this compensation plan, potential suppliers decide whether to enter. The assumption that potential suppliers can observe the parameters of the manager's compensation plan is not unreasonable. To avoid the pressures to expense employee stock options, firms that rely heavily upon them are becoming increasingly forthcoming about their use.

At the time when the owners of the buyer design the compensation scheme for their manager, the information about demand is represented by the density and distribution functions $g(x)$ and $G(x)$, and this information is available to the owners, the manager, and the potential suppliers. As previously discussed, after the suppliers observe the manager's compensation plan and determine whether to enter, some additional demand information becomes available. We denote this information by λ. Because this information is observable only by the manager, we do not consider the possibility that either the compensation plan or contracts with suppliers could be explicitly contingent upon the demand signal λ.

We incorporate the demand signal, λ, into our model in the following way: After observing the demand signal, the manager of the buying firm has the following conditional density and distribution functions for demand: $f_\lambda(x)$ and $F_\lambda(x)$, for which the range of positive support is $[d_{min}, d_{max}]$ for all λ, and $0 \leq d_{min} \leq d_{max}$. We assume that $F_\lambda(x)$ is a member of the family of Increasing Failure Rate distributions (IFR). Note that many common distributions satisfy the IFR property. In addition, we assume that $F_\lambda(x)$ is decreasing in λ, so that the demand is stochastically increasing in the value of the observed signal. Prior to the manager's observation of λ, all three parties (i.e. the owners, the manager, and the suppliers) have similar expectations about the relative likelihood of different demand signals. We characterize these expectations with a density function for the demand signal, $h(\lambda)$, for which the range of support is $[\lambda_{min}, \lambda_{max}]$. Thus, until the manager observes the realization of λ, all three parties' information about demand can be represented by:

$$g(x) = \int_{\lambda} f_{\lambda}(x)\, h(\lambda)\, d\lambda \quad \text{and} \quad G(x) = \int_{\lambda} F_{\lambda}(x)\, h(\lambda)\, d\lambda$$

In the special case that no new information becomes available between when the suppliers install capacity and when the manager of the buying firm determines her output, we have $f_{\lambda}(x) = f(x) = g(x)$ and $F_{\lambda}(x) = F(x) = G(x)$ for all λ. In such a case, the manager's role in observing the demand signal is eliminated. However, we will show that, even in this case, the manager can provide a useful role.

After observing the demand signal, the manager enters medium-term contracts with the suppliers who have entered. Recall that each of the suppliers faces an increasing marginal cost function, $s(q)$. Therefore, if the manager offers to buy the input at a per-unit procurement price of r, each supplier will respond by producing up to the point at which its marginal cost equals marginal revenue, i.e. $s(q) = r$. Thus, the per-unit procurement price that the manager would have to offer in order to obtain q units from each supplier would be $r = s(q)$, and the buying firm's total cost for a total output of Q units would be equal to $Q\left(c + s\left(\frac{Q}{n}\right)\right)$, where n is the number of suppliers in the market. Given the number of suppliers, n, output quantity, Q, and demand realization, x, the profit of the buying firm, before the manager gets paid, will be:

$$\pi_f(Q, n, x) = Min\{x, Q\}\, p - Q\left(c + s\left(\frac{Q}{n}\right)\right)$$

Note that both the manager, and her firm, accept some inventory risk by committing to quantity Q prior to the realization of demand. However, because this quantity is chosen only after the suppliers invest in capacity, there is no guarantee that they will recover their full costs. It is also worth noting that the output quantity, Q, that is chosen by the manager may depend upon how she is compensated. Under the assumption that suppliers can observe the manager's compensation plan and anticipate how she will respond, the manager's compensation package can influence the number of suppliers that enter.

Although we do not require that the manager's compensation be entirely incentive-based, we do require that her expected income from incentive based compensation be at least equal to some level, which we denote by $\overline{w}$, in order to insure participation. As the non-performance based compensation has no effect upon the manager's decisions, we do not show that term in our model. This is consistent with our interest in determining the appropriate form, rather than the amount, of incentive-based compensation. We consider a simple class of performance based compensation that is characterized by two parameters: $\alpha \in (0, 1)$ and R, such that the manager receives fraction α of the amount by which the firm's profit exceeds threshold R. Thus, at the time that the manager determines her output quantity, she will attempt to maximize her *conditional expected incentive compensation*, which can be represented as follows:

$$CEIC(Q; \alpha, R, n, \lambda) = E_{x|\lambda}\left[\alpha\left[\pi_f(Q, n, x) - R\right]^+\right] \tag{2.1}$$

where $[z]^+$ denotes $Max\,\{0, z\}$. Recall that we have assumed that the manager's output decision can be postponed until after the entry of the suppliers and the observation of demand signal λ, so that Q may depend upon the realization of λ.

Such a compensation scheme is similar in spirit to the granting of stock options, which gives the manager the option of acquiring ownership of fraction α of the firm's assets at a total exercise price of αR. A manager who holds such stock options will cash in the gain only when the stock price is above the exercise price. To the extent that a firm's market capitalization rises and falls with its current profit, our compensation scheme is similar to a scheme based on stock options. Note that, if the value of the firm were exactly equal to its current operating profits, then our proposed incentive compensation scheme would be equivalent to the granting of stock options. However, our model makes no attempt to establish a relationship between current profits and market expectations about future earnings which should be reflected in its market capitalization.

The expected value obtained by the original shareholders, which is equal to the expected profits of the firm less the manager's compensation, can be expressed as:

$$ESHV(R, \alpha) = E_\lambda \left[E_{x|\lambda} \left[\pi_f(Q, n, x) - \alpha \left[\pi_f(Q, n, x) - R \right]^+ \right] \right] \tag{2.2}$$

where Q depends on the compensation parameters (R, α), the number (n) of suppliers and the realization (λ) of the demand signal. Because suppliers can anticipate the manager's output decision, at equilibrium, suppliers will enter until it is no longer profitable to do so. Thus, the equilibrium number of suppliers will indirectly depend upon the incentive compensation parameters.

The owner's objective is to design an incentive based compensation plan to maximize $ESHV(R, \alpha)$ subject to the manager's participation constraint:

$$E_\lambda \left[CEIC(Q; \alpha, R, n, \lambda) \right] \geq \overline{w} \tag{2.3}$$

Because this participation constraint will be binding in any optimal solution to the share-holder's problem of designing a compensation plan, we can define $\alpha(R)$ to be the value of α that satisfies (2.3) at equality, and focus on identifying the optimal value of R. It should be noted that our approach implicitly assumes that the manager is risk neutral. Obviously this simplifies the mathematical presentation. However, this may be quite plausible when managers are guaranteed payoffs that make them financially secure for life regardless of how well they perform. For example, a number of top managers who were recently dismissed for poor performance (e.g. Carly Fiorina from Hewlett-Packard, Michael Eisner at Disney) received multi-million dollar severance packages. When the minimal level of compensation is already very large, risk neutrality is not a wholly unreasonable assumption.

2.4 The Benchmark Profit and Number of Suppliers

2.4.1 The Benchmark Solution

To obtain a benchmark, it is useful to consider the number (n) of suppliers that would enter to serve the market if the manager of the buyer could enter long-term contingent contracts with suppliers. In this context, to enter long-term contingent contracts would involve specifying a price and a quantity that would be procured from each supplier for each possible realization of the demand signal λ. Obviously, such contracts are not possible since λ is observable only to the manager and cannot be verified by the suppliers. Nevertheless, it is useful for us to analyze this case because, if it were possible to contract fully, the buyer could obtain the first best level of capacity and output.

Let us begin by considering a compensation scheme that motivates the manager to maximize the expected operating profit, $E_\lambda\left[E_{x|\lambda}\left[\pi_f(Q,n,x)\right]\right]$, of the firm. For example, the manager would have such motivation if her incentive compensation were linearly increasing in the profits of the firm.

As a benchmark, let us suppose that it were possible for the manager to enter long-term contingent contracts with suppliers. Depending upon the number n of suppliers with whom the manager contracts, once the demand signal λ is observed, the combined profits of the suppliers and the buyer can be expressed as the following function of output for $Q \geq d_{min}$:

$$\pi^{SC}(Q; n, \lambda) = p\left(d_{min} + \int_{d_{min}}^{Q}(1 - F_\lambda(x))dx\right) - n\int_0^{Q/n}(c + s(q))dq \quad (2.4)$$

where, because of the convex marginal costs, it is optimal to allocate production evenly across these suppliers. Thus the marginal cost to the supply chain for producing the Q^{th} unit is $c + s\left(\frac{Q}{n}\right)$.

By taking first and second derivatives, it is easy to confirm that $\pi^{SC}(Q; n, \lambda)$ is concave, and that it is maximized when $Q = Q^{FB}(n, \lambda) = \bar{F_\lambda}^{-1}\left(\frac{c+s(Q/n)}{p}\right)$, where $\bar{F_\lambda}^{-1}$ denotes the inverse of the converse cumulative of the conditional demand distribution. Note that $Q^{FB}(n, \lambda)$ represents the *first best* quantity that maximizes the combined profits of the buyer and her suppliers given that there are n suppliers and the demand signal is λ. To avoid the boundary condition, we assume that $c + s(d_{min}) < p$, so that we will have $Q^{FB}(n, \lambda) > d_{min}$.

Under the assumption that the supply industry is competitive and open to entry, a long-term contingent contract would need to guarantee each supplier a non-negative expected profit. Thus, when the manager determined the number of suppliers with whom to contract, maximizing the profit of her firm would be consistent with maximizing the total supply chain profit, which can be expressed as a function of n:

$$\pi^{SC}(n) \quad = \quad -Kn + \int_{\lambda} \pi^{SC}(Q^{FB}; n, \lambda) h(\lambda) d\lambda$$

For analytical tractability, we will treat n as a continuous variable throughout the paper. It can be shown that $\frac{d^2}{dn^2}\pi^{SC}(Q^{FB}; n, \lambda) \leq 0$. Therefore, since any convex combination of concave functions is also concave, $\pi^{SC}(n)$ must also be concave. Define n^{FB} to be the number of suppliers that maximize $\pi^{SC}(n)$.

Note that if the manager could enter long-term contingent contracts with suppliers, any incentive-based compensation plan that would give the manager an expected incentive compensation of $\overline{w}$ and would encourage her to maximize expected profits would be optimal from the perspective of the owners. For all such plans, the manager's expected compensation would be $\overline{w} = \alpha \left(E\left[\pi^{SC}(n^{FB})\right] - R \right)$ and the expected shareholder value would be: $E\left[\pi^{SC}(n^{FB})\right] - \overline{w}$.

2.4.2 The Benchmark Solution under Special Case of No Demand Signal

In the special case in which the signal λ provides no new information about demand, i.e. $F_{\lambda}(x) = F(x)$, the manager of the buyer simultaneously determines the number (n) of suppliers with which to contract and the level of output $\left(\frac{Q}{n}\right)$ that it will procure from each one. For this situation, the expected profits can be maximized by operating each supplier at the most efficient level and contracting with just enough suppliers to produce its chosen level of output. Recall that we are treating the number (n) of suppliers as a continuous variable. Given our assumptions that each supplier incurs a fixed investment cost of \$$K$ and has convex marginal costs

$s(q)$ that are increasing beyond some threshold level of output, it follows there exists some volume of output, $\hat{q} \geq 0$, at which a supplier's average variable costs are minimized and some higher level of output, $q^o \geq \hat{q}$, at which its average total costs are minimized. Define $\hat{s} = s(\hat{q})$ and $s^o = s(q^o)$. This notation is borrowed from Subramaniam (1998). Hence, the optimal number of suppliers with which the manager would contract and the total level of output would satisfy:

$$n^{FB} = \frac{Q^{FB}}{q^o} \quad \text{and} \quad Q^{FB} = \bar{F}^{-1}\left(\frac{c + s^o}{p}\right)$$

In this special case where the demand signal provides no information, we need not worry about the lack of verifiability interfering with the firm's ability to contract. However, enforceability could still be a problem. For example, as described in Dyer et al. (1998), General Motors saved roughly \$3-4 billion in the early 1990's as a result of opening up existing supply contracts to competitive bidding. However, as a consequence of these hard-ball tactics, General Motors subsequently had difficulty convincing suppliers to invest in developing technology and capacity that were specific to its products. Critics contend that General Motors continues to feel the effects of their suppliers' lack of confidence that the terms of a contract will be honored. Clearly, the ability of a long term contract to eliminate the hold-up problem is limited by the extent to which the suppliers believe that the contract will be honored and enforced.

2.5 Non-Contractability and Managerial Compensation

2.5.1 Equilibrium Outcomes in the Absence of Pre-Commitment

When the manager cannot enter long-term contingent contracts with her suppliers, she no longer has direct control over the number of suppliers who enter. Instead, suppliers will enter the industry so long as they anticipate earning non-negative expected profits. Recall that at the time that the manager determines her output quantity, she must offer to pay a per-unit price of $r = s(q)$ in order to induce each supplier to produce q units. Thus, at the time that the manager of the buying firm makes her output decision, she will seek to maximize the conditional expected incentive compensation, $CEIC(Q; \alpha, R, n, \lambda)$, as defined in (2.1).

Recall that if it were possible to enter long-term contingent contracts with suppliers, any compensation plan that encouraged the manager to seek to maximize the operating profits would be sufficient to maximize the total value of the shareholders. Let us now consider how such a profit maximization compensation plan affects the buyer's financial outcome when she cannot enter such contingent contracts with suppliers. After observing the number of suppliers and the demand signal, the manager will determine the output quantity Q in order to maximize the conditional expected operating profit of the firm, which can be expressed as follows:

$$E_{x|\lambda}\left[\pi_f(Q; n, x)\right] \;=\; p\left(d_{min} + \int_{d_{min}}^{Q} (1 - F_\lambda(x))dx\right) - Q\left(c + s\left(\frac{Q}{n}\right)\right) \quad (2.5)$$

It is easy to show that $E_{x|\lambda}\left[\pi_f(Q; n, x)\right]$ is concave in Q. Let $Q^{PM}(n, \lambda)$ denote the volume of output for which the conditional expected operating profit

$E_{x|\lambda}\left[\pi_f(Q;n,x)\right]$ is maximized (The superscript "PM" stands for profit maximization).

Proposition 1 *For a given number n of suppliers, and demand signal λ, a profit maximizing manager will choose a smaller level of output if she cannot enter long-term contingent contracts with suppliers than if she can. Formally: $Q^{PM}(n,\lambda) < Q^{FB}(n,\lambda)$.*

Proof: Recall that $Q^{PM}(n,\lambda)$ is the level of output that maximizes (2.5). It is easy to confirm that (2.5) is concave and so $Q^{PM}(n,\lambda)$ is the value of Q that satisfies the following first order condition:

$$p\overline{F}_\lambda(Q^{PM}) - \left(c + s\left(\frac{Q^{PM}}{n}\right)\right) - \frac{Q^{PM}}{n}s'\left(\frac{Q^{PM}}{n}\right) = 0 \tag{2.6}$$

Recall that $Q^{FB}(n,\lambda)$ is the first best output level for the supply chain optimization problem defined by (2.4). By differentiating (2.4) with respect to Q, we derive the following first order condition that $Q^{FB}(n,\lambda)$ has to satisfy:

$$p\overline{F}_\lambda(Q^{FB}) - \left(c + s\left(\frac{Q^{FB}}{n}\right)\right) = 0 \tag{2.7}$$

Since $s(\cdot)$ is increasing and convex, it follows that:

$$p\overline{F}_\lambda(Q^{FB}) - \left(c + s\left(\frac{Q^{FB}}{n}\right)\right) < p\overline{F}_\lambda(Q^{PM}) - \left(c + s\left(\frac{Q^{PM}}{n}\right)\right)$$

As $p\overline{F}_\lambda(Q) - \left(c + s\left(\frac{Q}{n}\right)\right)$ decreases with Q, it follows that $Q^{PM}(n,\lambda) < Q^{FB}(n,\lambda)$. $\diamondsuit$

Recall that suppliers will enter the market as long as they can earn non-negative expected profits from supplying the anticipated volumes ordered by the buyer. Let

n^{PM} denote the number of suppliers that enter in equilibrium when it is anticipated that the manager's output decision will be $Q^{PM}(n, \lambda)$. The total combined expected profits for the buyer and its profit maximizing manager, can be represented as follows:

$$\pi^{PM}(n^{PM}) \;\;=\;\; \int_{\lambda} \left(E_{x|\lambda} \left[\pi_f(Q^{PM}; n^{PM}, x) \right] \right) h(\lambda) d\lambda$$

So long as the managerial compensation plan specifies R sufficiently small such that $\pi_f(Q^{PM}(n^{PM}, \lambda); n^{PM}, x) \geq R$ for even the smallest realizations of demand, at equilibrium we will have n^{PM} suppliers and the manager will choose an output level of $Q^{PM}(n^{PM}, \lambda)$.

As was the case when contracts could be used, the owners will be indifferent among all compensation plans that satisfy the participation constraint at equality and encourage the manager to maximize profits. For any of these plans, the manager's expected incentive-based compensation is $E_\lambda [CEIC(Q; \alpha, R, n, \lambda)] = \overline{w}$ and the expected final shareholder value is: $\pi^{PM}(n^{PM}) - \overline{w}$.

Corollary 1 *In the absence of any pre-commitment from the buying firm, only* $n^{PM} < n^{FB}$ *suppliers will enter and the buying firm's profit will be less than what she could earn had* n^{FB} *suppliers enterred, i.e.* $\pi^{PM}(n^{PM}) < \pi^{SC}(n^{FB}) < \pi^{PM}(n^{FB})$.

This corollary follows from the fact that suppliers will enter only so long as they anticipate that the expected output decision of the buyer will allow them to cover their expected fixed and variable costs. Recall that, in order for n^{FB} suppliers to enter, they must anticipate that the buyer's output quantity will be $Q^{FB}(n, \lambda)$. If n^{FB} suppliers enter, and the buyer's output quantity is $Q^{FB}(n^{FB}, \lambda)$, then in expectation, the suppliers break even and the buyer's profit is equal to $\pi^{SC}(n^{FB})$, i.e. the maximum profit that she could earn under contingent contracting. However, in the absence of contingent contracting, it is clear from Proposition 1 that if n^{FB} suppliers were to enter, the buyer could increase her own profits by setting her output quantity lower than the first best quantity, i.e. increasing her own profits at the expense of the suppliers. In anticipation of this, fewer than the efficient number of suppliers will enter to serve the buyer. Moreover, since the buyer's profits are increasing in the number of suppliers and $\pi^{SC}(n^{FB})$ is the first best supply chain profit, it follows that $\pi^{PM}(n^{PM}) < \pi^{SC}(n^{FB}) < \pi^{PM}(n^{FB})$.

2.5.2 Equilibrium Outcomes under Performance-Based Compensation

2.5.2.1 Performance-Based Compensation with Threshold

In the previous section, we demonstrated that, if the manager of the buyer has an incentive to maximize operating profits and she is unable to enter long-term contingent contracts with suppliers, then fewer than the efficient number of suppliers will invest in entering the industry. Recall that under our class of compensation schemes, the manager's performance based compensation is equal to $\alpha\left[(\pi_f(Q;n,x) - R)\right]^+$. Previously, we assumed that the threshold of the compensation plan, R, was sufficiently small such that $\pi_f(Q^{PM}(n^{PM},\lambda); n^{PM}, x) - R > 0$ for even the smallest demand realizations. We now consider performance threshold values that are sufficiently large that there is a positive probability that the manager will receive zero performance based compensation.

To understand how the manager's compensation will affect the ultimate value of the holdings of the firm owners, we must first characterize how the manager will make output decisions, conditional upon n and λ. Let us define $MFV(Q,n) =$

$pQ - Q\left(c + s\left(\frac{Q}{n}\right)\right)$ to be the maximum profit of the firm given that there are n suppliers, and the manager sets the output to Q. Obviously, this maximum firm value is realized only when the demand realization is no smaller than Q. Since $MFV(Q,n)$ is concave in Q, for every n, there is a unique value, $Q^{MFV}(n)$, for which it is maximized. Define $MFV^*(n) = MFV\left(Min\left\{Q^{MFV}(n), d_{max}\right\}, n\right)$ to be the maximum possible value of the firm when there are n suppliers.

Obviously, if the performance threshold $R \geq MFV^*(n)$, then the performance-based compensation is of no value to the manager and does not affect her output decision. We therefore focus on the case where $R < MFV^*(n)$, for which, due to the concavity of the function $MFV(Q,n)$, there exists a range $\left[\underline{Q}(R,n), \bar{Q}(R,n)\right]$, such that for any $Q \in \left[\underline{Q}(R,n), \bar{Q}(R,n)\right]$, we have $MFV(Q,n) \geq R$. Note that $MFV(Q,n)$ is the upper bound on the actual operating profits of the buyer when the manager sets the output level at Q and n suppliers are in the market. The realized profits will be $MFV(Q,n)$ only when the demand turns out to be at least Q. Thus, only when the manager chooses a quantity in the range $\left[\underline{Q}(R,n), \bar{Q}(R,n)\right]$, can buyer's actual operating profits exceed R under some demand realizations to yield a positive amount of incentive compensation, i.e. for the manager. As a result, the manager who tries to maximize her payoffs will always prescribe a Q in this range.

Moreover, for any $Q \in \left[\underline{Q}(R,n), \bar{Q}(R,n)\right]$, there exists a minimum realization of demand $d(Q,R,n)$, above which the actual profit is at or above R. Let us formally define this minimum realization of demand as follows:

$$d(Q,R,n) = Max\left\{d_{min}, \frac{R + Q\left(c + s\left(Q/n\right)\right)}{p}\right\} \tag{2.8}$$

Obviously, $d(Q,R,n)$ is increasing in Q and R, and is decreasing in n, for $d(Q,R,n) \geq d_{min}$. In addition, it is straightforward to show that $d(Q,R,n) \leq Q$ for any

$Q \in \left[\underline{Q}(R,n), \bar{Q}(R,n)\right]$.

Note that we can also verify that If $R < MFV^*(n)$, then the manager's conditional expected incentive-based compensation at the time she makes her output decision can be expressed as follows:

$$CEIC(Q;\alpha,R,n,\lambda) = \alpha \int_{d(Q,R,n)}^{d_{max}} \left(pMin\{x,Q\} - Q\left(c + s\left(\frac{Q}{n}\right)\right) - R\right) f_\lambda(x)dx \tag{2.9}$$

To determine how the manager's incentive compensation affects her decision, recall that $Q^{PM}(n,\lambda)$ is the quantity that the manager would produce if she were trying to maximize the total operating profits. Let $\hat{R}(n,\lambda)$ be the maximum value of the manager's compensation parameter, for which she would be guaranteed some incentive pay for any realization of demand, given that there are n suppliers and that the manager orders $Q^{PM}(n,\lambda)$ units in response to demand signal λ. Formally, $\hat{R}(n,\lambda)$ is the value of R for which $d\left(Q^{PM}(n,\lambda), R, n\right) = d_{min}$ or:

$$\hat{R}(n,\lambda) = pd_{min} - Q^{PM}(n,\lambda)\left(c + s\left(\frac{Q^{PM}(n,\lambda)}{n}\right)\right) \tag{2.10}$$

Proposition 2 *a) For any $R < MFV^*(n)$, the manager's conditional expected incentive compensation, $CEIC(Q;\alpha,R,n,\lambda)$ is unimodal in her output quantity within the range $Q \in [\underline{Q}(R,n), \bar{Q}(R,n)]$.*

b) The manager's optimal output can be characterized as follows: If $R \leq \hat{R}(n,\lambda)$, then $Q^*(R,n,\lambda) = Q^{PM}(n,\lambda)$. If $R > \hat{R}(n,\lambda)$, then $Q^*(R,n,\lambda)$ is the value of Q that satisfies:

$$p\bar{F}_\lambda(Q) - \bar{F}_\lambda(d(Q,R,n))\left(c + s\left(\frac{Q}{n}\right) + \frac{Q}{n}s'\left(\frac{Q}{n}\right)\right) = 0 \tag{2.11}$$

and we will have $Q^*(R,n,\lambda) > Q^{PM}(n,\lambda)$ *and* $\frac{\partial Q^*(R,n,\lambda)}{\partial R} > 0$.

Proof:

a) To show that $CEIC(Q;\alpha,R,n,\lambda)$ is unimodal, it is helpful to work with the following expression:

$$
\begin{aligned}
&CEIC(Q;\alpha,R,n,\lambda)\\
&\qquad =\alpha\int_{d(Q,R,n)}^{d_{max}}\left(p\times Min\left\{Q,x\right\}-(c+s(\frac{Q}{n}))Q-R\right)f_\lambda(x)dx
\end{aligned}
\tag{2.12}
$$

where $d(Q,R,n)$ is as defined in (2.8). It suffices to show that (2.12) is locally concave at any point that satisfies the first-order condition. Let $Q^o(R,n)$ be the minimum value of Q for which $d(Q,R,n)\geq d_{min}$. Differentiating (2.12) with respect to Q, we have:

$$
\begin{aligned}
&\frac{dCEIC(Q;\alpha,R,n,\lambda)}{dQ}\\
&=\begin{cases} p\bar{F}_\lambda(Q)-\left(c+s\left(\frac{Q}{n}\right)+\frac{Q}{n}s'\left(\frac{Q}{n}\right)\right) & for\ Q\leq Q^o(R,n)\\ p\bar{F}_\lambda(Q)-\bar{F}_\lambda(d(Q,R,n))\left(c+s\left(\frac{Q}{n}\right)+\frac{Q}{n}s'\left(\frac{Q}{n}\right)\right) & for\ Q\geq Q^o(R,n)\end{cases}
\end{aligned}
\tag{2.13}
$$

Differentiating again, we have:

$$
\begin{aligned}
&\frac{d^2CEIC(Q;\alpha,R,n,\lambda)}{dQ^2}\\
=\ & -pf_\lambda(Q)-\bar{F}_\lambda(d(Q,R,n))\left(\frac{Q}{n^2}s''\left(\frac{Q}{n}\right)+\frac{2}{n}s'\left(\frac{Q}{n}\right)\right)\\
&+f_\lambda(d(Q,R,n))\frac{\delta d(Q,R,n)}{\delta Q}\left(c+s\left(\frac{Q}{n}\right)+\frac{Q}{n}s'\left(\frac{Q}{n}\right)\right)
\end{aligned}
\tag{2.14}
$$

It follows from the definition of $d(Q,R,n)$ that $d(Q,R,n) = d_{min}$ for $Q \leq Q^o(R,n)$, while $d(Q,R,n)$ is continuous and increasing for $Q \geq Q^o(R,n)$ within the range $[\underline{Q}(R,n), \bar{Q}(R,n)]$. Observe that the first two terms in (2.14) are both non-positive. For $Q \leq Q^o(R,n)$, we have that $\frac{\delta d(Q,R,n)}{\delta Q} = 0$, so that $\frac{d^2CEIC(Q;\alpha,R,n,\lambda)}{dQ^2} < 0$ and $CEIC(Q;\alpha,R,n,\lambda)$ must be concave, hence unimodal, in this range. Now, for $Q \geq Q^o(R,n)$, we have that $\frac{\delta d(Q,R,n)}{\delta Q} = \frac{1}{p}\left(c + s\left(\frac{Q}{n}\right) + \frac{Q}{n}s'\left(\frac{Q}{n}\right)\right)$. Noting that the second term in (2.14) is negative, we can substitute for $\frac{\delta d(Q,R,n)}{\delta Q}$ to obtain:

$$\frac{d^2CEIC(Q;\alpha,R,n,\lambda)}{dQ^2} \quad < \quad -pf_\lambda(Q) + \frac{\left(c + s\left(\frac{Q}{n}\right) + \frac{Q}{n}s'\left(\frac{Q}{n}\right)\right)^2 f_\lambda(d(Q,R,n))}{p}$$

$$\begin{aligned} &= \frac{\bar{F}_\lambda(Q)\bar{F}_\lambda(d)}{p}\left(\frac{-p^2 f_\lambda(Q)}{\bar{F}_\lambda(Q)\bar{F}_\lambda(d)} + \frac{\left(c + s\left(\frac{Q}{n}\right) + \frac{Q}{n}s'\left(\frac{Q}{n}\right)\right)^2 f_\lambda(d)}{\bar{F}_\lambda(Q)\bar{F}_\lambda(d)}\right) \\ &= \bar{F}_\lambda(Q)\left(\frac{-pf_\lambda(Q)}{\bar{F}_\lambda(Q)} + \frac{\left(c + s\left(\frac{Q}{n}\right) + \frac{Q}{n}s'\left(\frac{Q}{n}\right)\right) f_\lambda(d)}{\bar{F}_\lambda(d)}\right) \end{aligned} \qquad (2.15)$$

where the latter equality follows from (2.13), which implies that the following equation holds:

$$\frac{p}{\bar{F}_\lambda(d)} = \frac{1}{\bar{F}_\lambda(Q)}\left(c + s\left(\frac{Q}{n}\right) + \frac{Q}{n}s'\left(\frac{Q}{n}\right)\right)$$

Recall that for any $Q \in \left[\underline{Q}(R,n), \bar{Q}(R,n)\right]$, $d(Q,R,n) \leq Q$. To determine the sign of the right hand side of (2.15), we can again use the first order condition (2.13) and the fact $d \leq Q$ to show that $p \geq c + s\left(\frac{Q}{n}\right) + \frac{Q}{n}s'\left(\frac{Q}{n}\right)$. From the fact that $d \leq Q$ and the assumption that $f_\lambda(\cdot)$ is IFR, we have $\frac{f_\lambda(Q)}{\bar{F}_\lambda(Q)} > \frac{f_\lambda(d)}{\bar{F}_\lambda(d)}$. It follows that (2.15) is negative at any point satisfying the first order condition.

Finally, since $\bar{F}_\lambda(d(Q,R,n)) = 1$ for $Q = Q^o(R,n)$, it follows that $\frac{dCEIC(Q;\alpha,R,n,\lambda)}{dQ}$ is continuous. Hence, first-order conditions are sufficient to define the manager's optimal output quantity in response to threshold R for any given values of n and λ.

b) From (2.10) it can be seen that $Q^o(R,n) = Q^{PM}(R,n)$ when $R = \hat{R}(n,\lambda)$. When $R \leq \hat{R}(n,\lambda)$, then $Q^o(R,n) \geq Q^{PM}(R,n)$. It follows from the continuity condition that the first order condition for $CEIC(Q;\alpha,R,n,\lambda)$ is defined by setting the top branch of (2.13) equal to zero, and it is satisfied at the point $Q = Q^{PM}$. When $R > \hat{R}(n,\lambda)$, then $Q^o(R,n) < Q^{PM}(R,n)$, and the first order condition for $CEIC(Q;\alpha,R,n,\lambda)$ is defined by setting the lower branch of (2.13) equal to zero, as shown in (2.11). By comparison of the lower and upper branch of (2.13), it follows from the facts that $d(Q,R,n) > d_{min}$ and $s(\cdot)$ is convex increasing that $Q^*(R,n,\lambda) > Q^{PM}(n,\lambda)$ when $R > \hat{R}(n,\lambda)$.

To determine the sign of $\frac{\partial Q^*(R,n,\lambda)}{\partial R}$, differentiating the lower branch of (2.13), with respect to the incentive compensation parameter, R, we have:

$$\begin{aligned} -pf_\lambda(Q^*)\frac{\partial Q^*}{\partial R} - \left(\frac{2}{n}s'\left(\frac{Q^*}{n}\right)\frac{\partial Q^*}{\partial R} + \frac{Q^*}{n^2}s''\left(\frac{Q^*}{n}\right)\frac{\partial Q^*}{\partial R}\right)\bar{F}_\lambda(d^*) & \\ + \left(c + s\left(\frac{Q^*}{n}\right) + \frac{Q^*}{n}s'\left(\frac{Q^*}{n}\right)\right)f_\lambda(d^*)\frac{\partial d^*}{\partial R} &= 0 \end{aligned}$$

where, for ease of exposition, we have omitted the functional parameters of $Q^*(R,n,\lambda)$ and refer to $d(Q^*(R,n,\lambda),R,n)$ by d^*. Because $d^* > d_{min}$, by implicit differentiation of $d^* = \frac{R+Q(c+s(Q/n))}{p}$, we have $\frac{\partial d^*}{\partial R} = \frac{1}{p} + \frac{(c+s(\frac{Q^*}{n})+\frac{Q^*}{n}s'(\frac{Q^*}{n}))}{p}\frac{\partial Q^*}{\partial R}$. Substituting into above equation, we have:

$$\begin{aligned} &\frac{\partial Q^*}{\partial R}\left[-pf_\lambda(Q^*) - \left(\frac{2}{n}s'\left(\frac{Q^*}{n}\right) + \frac{Q^*}{n^2}s''\left(\frac{Q^*}{n}\right)\right)\bar{F}_\lambda(d^*) + \frac{\left(c + s\left(\frac{Q^*}{n}\right) + \frac{Q^*}{n}s'\left(\frac{Q^*}{n}\right)\right)^2 f_\lambda(d^*)}{p}\right] \\ &+ \left(c + s\left(\frac{Q^*}{n}\right) + \frac{Q^*}{n}s'\left(\frac{Q^*}{n}\right)\right)\frac{f_\lambda(d^*)}{p} = 0 \end{aligned} \tag{2.16}$$

The part inside the square brackets in the LHS is the second derivative of manager's expected incentive compensation at optimal output level Q^*. As the expected compensation is unimodal, this term must be non-positive. The second term in the equation (2.16) is strictly positive as $s(q)$ is increasing and convex. Thus we must have $\frac{\partial Q^*(n,R,\lambda)}{\partial R} > 0$. $\diamondsuit$

It follows from part *b)* that $\frac{\partial Q^*(R,n,\lambda)}{\partial R} = 0$ for $R < \hat{R}(n, \lambda)$, so that very low performance thresholds have no marginal effect upon the manager's decision. In this range, the manager always seeks to maximize the profit of her firm. On the other hand, when $R > \hat{R}(n, \lambda)$, we have $d(Q^*(n, R), R, n) > d_{min}$. This result confirms that if the incentive compensation performance threshold is sufficiently high, then the manager will not receive incentive compensation for all demand realizations. As a consequence, the manager will be insensitive to low realizations of demand and will find it in her interest to increase her output quantity. That also means the manager's objective is no longer consistent with maximizing the expected operating profits of her firm. Moreover, her output quantity will be increasing in the incentive compensation performance parameter. As R increases, the manager becomes more and more aggressive and orders a higher level of output.

2.5.2.2 Supplier's Entry Decision

In order to understand the relationship between the incentive compensation parameter R and the number n of suppliers, we must consider the suppliers' entry decision. At the time of entry, suppliers will anticipate the manager's output quantity, $Q^*(R, n, \lambda)$, as characterized in Proposition 2. Thus, for a given number of suppliers and demand signal, they will anticipate that the manager of the firm will offer to buy their input for $s\left(\frac{Q^*(R,n,\lambda)}{n}\right)$. The expected profit of each supplier conditional upon the number of suppliers that enter can be expressed as:

$$\pi_s(K,R,n)$$
$$= -K + \int_{\lambda}\left(\frac{Q^*(R,n,\lambda)}{n}s\left(\frac{Q^*(R,n,\lambda)}{n}\right) - \int_0^{\frac{Q^*(R,n,\lambda)}{n}} s(y)dy\right)h(\lambda)d\lambda \quad (2.17)$$

Recall that, to facilitate the analysis, we treat n as a continuous variable. It follows that suppliers will enter until $\pi_s(K,R,n) = 0$. Denote the equilibrium number of suppliers by $n^*(R)$. In addition, let us now define $\hat{R}(n)$ to be the smallest value of $\hat{R}(n,\lambda)$ for any demand signal that can be observed. It can be shown from (2.6) that $Q^{PM}(n,\lambda)$ is increasing in λ and from (2.10) that $\hat{R}(n,\lambda)$ is decreasing in $Q^{PM}(n,\lambda)$. Thus, $\hat{R}(n) = \hat{R}(n,\lambda_{max})$. We also define $MFV^* = max_{Q,n}\{MFV(Q,n)|\, Q = nq^o\}$, which is the maximum profit that can be reached under large demand realizations when the buyer demands exactly q^o from each supplier.

Proposition 3 *For $R \leq \hat{R}(n^{PM})$, the equilibrium number of suppliers to enter will be $n^*(R) = n^{PM}$, while for $R \in (\hat{R}(n^{PM}), MFV^*)$, we will have $n^*(R) > n^{PM}$ and $n^*(R)$ increasing in R.*

Proof:

a) When $R \leq \hat{R}(n^{PM})$, we have from Proposition 2 that $Q^*(R,n,\lambda) = Q^{PM}(n,\lambda)$. In anticipation of this, suppliers will enter until the point at which $\frac{Q^*(n,\lambda)}{n} = \frac{Q^{PM}(n,\lambda)}{n} = q^o$. By definition, the number of suppliers to enter will be $n^*(R) = n^{PM}$.
b) When $R > \hat{R}(n^{PM})$, we have from Proposition 2 that $Q^*(R,n,\lambda) > Q^{PM}(n,\lambda)$. Because, suppliers will enter until the point at which $\frac{Q^*(n,\lambda)}{n} = q^o$, it follows that $n^* > n^{PM}$.

For $R > \hat{R}(n^{PM})$, an implicit relationship between R and n is defined by the equation $\pi_s(K,R,n) = 0$ where $\pi_s(K,R,n)$ is as shown in (2.17). Totally differentiating the equation with respect to R, we have:

$$\begin{aligned}\frac{d\pi_s(K,R,n)}{dR} &= \frac{\partial \pi_s(K,R,n)}{\partial Q^*}\frac{\partial Q^*}{\partial R} + \frac{\partial \pi_s(K,R,n)}{\partial n}\frac{\partial n}{\partial R} \\ &= \left(\int_\lambda \frac{Q^*}{n^2} s'\left(\frac{Q^*}{n}\right) h(\lambda)d\lambda\right)\frac{\partial Q^*}{\partial R} - \left(\int_\lambda \frac{Q^*}{n^3} s'\left(\frac{Q^*}{n}\right) h(\lambda)d\lambda\right)\frac{\partial n}{\partial R} = 0\end{aligned}$$

Because marginal costs are increasing, and we know from Proposition 2 that $\frac{\partial Q}{\partial R} > 0$, it follows that $\frac{\partial n}{\partial R} > 0$. $\diamondsuit$

This result confirms the fact that by placing a threshold on the manager's performance based compensation, so that she receives nothing under some low realizations of demand, the owners of the firm can credibly commit to higher levels of output and thereby encourage the entry of suppliers. Note that any performance threshold R that is larger than MFV^* cannot be reached under any situation and thus any meaningful value for R should be smaller than MFV^*.

2.5.2.3 The Optimal Incentive Plan

At the time that the owners of the firm design the incentive compensation scheme for their manager, they attempt to maximize the expected final value of their holdings subject to the requirement that the manager's expected incentive-based compensation be at least $\overline{w}$. Thus, they attempt to maximize $ESHV(R, \alpha)$, as shown in (2.2) subject to the manager's participation constraint in (2.3). Since the manager's participation constraint will be binding in any optimal solution, we can incorporate the constraint into the owner's objective function as follows:

$$ESHV(R) = -\overline{w} + \int_{\lambda}\int_{d_{min}}^{d_{max}} (pMin\{Q^*(R, n^*(R), \lambda), x\}) f_{\lambda}(x) dx h(\lambda) d\lambda$$
$$- \int_{\lambda} Q^*(R, n^*(R), \lambda)\left(c + s\left(\frac{Q^*(R, n^*(R), \lambda)}{n^*(R)}\right)\right) h(\lambda) d\lambda \quad (2.18)$$

Proposition 4 *For the special case in which the signal λ provides no new information about demand, i.e. $F_{\lambda}(x) = F(x)$ for all λ, the optimal solution to the shareholders' compensation design problem will have $R^* > \hat{R}(n^{PM})$. In equilibrium, n^{FB} suppliers will enter, and the manager will set the output quantity to $Q^{FB}(n^{FB}, \lambda)$.*

The above result follows from the observation that, as expressed in (2.18), the shareholder's expected final value is maximized when $n^*(R) = n^{FB}$, and $Q^*(R, n^*(R), \lambda) = Q^{FB}(n^{FB}, \lambda)$, both of which will occur when $F_{\lambda}(x) = F(x)$ for all λ and R is chosen appropriately to induce the first-best number of suppliers to enter.

The proposition demonstrates that when no useful demand information is revealed between the entry of the suppliers and when the manager of the buying firm determines her output quantity, the supplier holdup problem can be completely mitigated by an appropriate performance based compensation scheme in which the manager is rewarded only when profits exceed a certain threshold level. Moreover, the owners of the buying firm will find it in their own best interest to offer the compensation scheme that completely mitigates supplier hold-up. By doing so, they are able to provide a credible commitment to a higher level of output than would be in their own best interest after the entry of the efficient number of suppliers.

Note that in this special case, where the demand signal provides no useful information, the manager's nominal role of interpreting and responding to the demand signal is eliminated. However, by handing the reins of control over to a manager who is appropriately compensated, the owners of the firm can commit to a higher level of output. Recall that once the suppliers have entered the market, a profit maximizing manager would have an incentive to produce less than the first best quantity. In anticipation of this, fewer than the efficient number of suppliers would enter. The optimal performance based compensation scheme works precisely because it leads the manager of the firm to make an output decision that is larger than the one that would maximize the profits of the firm subsequent to the entry of the first-best number of suppliers.

In reality, it is more likely that some demand information λ will be revealed between the entry of suppliers and when the manager of the buyer must commit to her output quantity. When λ turns out to be small, the manager produces a small amount of output, under which the suppliers will not recover their sunk investment. When λ is big, the manager orders a large quantity from the suppliers. As a consequence, each supplier's expected gain is strictly positive. Nonetheless, the incentive scheme still plays an important role in determining the outcomes. A higher performance threshold will lead to a higher output and a larger number of suppliers.

Proposition 5 *If the signal λ provides useful information about demand, the optimal solution to the shareholders' compensation design problem will have $R^* > \hat{R}(n^{PM})$. In equilibrium, $n^{PM} < n^* < n^{FB}$ suppliers will enter, and the manager will set the output quantity between $(Q^{PM}(n^{PM}, \lambda), Q^{SC}(n^{FB}, \lambda))$.*

Under the optimal incentive scheme, the holdup problem can be at least partially mitigated. In order to encourage suppliers to enter, the owners will select a pay scheme that leads to incentive differences between the manager and themselves.

2.6 Concluding Remarks

We have studied the use of the performance based compensation as a vehicle to mitigate the holdup problem that arises when suppliers' relationship-specific investments in capacity leave them vulnerable to subsequent exploitation by a downstream supply chain partner. The possibility of being held up discourages the suppliers from investing to an optimal level. We show that, by implementing a performance based compensation scheme that rewards the manager only when firm profits are above a certain threshold, the owners can create an incentive for the manager to choose higher levels of output than those that would maximize firm profits. However, because suppliers anticipate these higher levels of output, more of them enter the market. As a result, such a performance based compensation scheme can benefit owners.

For situations in which no new demand information is revealed between the entry of suppliers and the time at which the manager determines her output quantity, we have shown that the optimal threshold based compensation scheme, from the owners perspective, completely mitigates the hold-up problem. Thus, for this special case, threshold based compensation is completely substitutable for long-term supply contracts. More generally, when partial demand information can be observed subsequent to supplier entry, we have shown that threshold based compensation can play an important role in improving the profits of the buying firm by inducing more entry from suppliers. Thus, a threshold based compensation scheme for a firm's management can provide at least a partial substitute for a long-term contingent contracts with suppliers. This is particularly important when issues of observability, verifiability, enforcement or other issues rule out long-term contingent contracts with suppliers. Although we have not considered other forms of long-term contracts with suppliers, e.g. take-or-pay agreements, subsidization of fixed costs, etc., we note that many firms avoid such contracts as a matter of policy. Since many of these firms also use employee stock options or other forms of threshold incentive compensation, it is of interest to understand whether these managerial compensation schemes can help to mitigate the supplier hold-up problem.

There are two important limitations to our model. First, we have assumed that the manager is risk neutral. Although we argue that this is not unreasonable in cases in which even the manager's minimum compensation is very large, it would certainly be of interest to study how the firm and the suppliers make their decisions if the manager's risk preference changes. We have also ignored the effect of the performance based compensation scheme upon the manager's level of effort. Since this is the more traditional framework for analyzing performance based compensation, it would be useful to consider how a compensation scheme should strike a balance between encouraging the manager to exert effort and mitigating the supplier holdup problem. Empirical studies are also needed to verify the signaling effects of the suggested incentive plan to the suppliers. All these are avenues for future research.

Chapter 3

Technology Licensing under Complementary Effects

3.1 Introduction

Many innovative technologies involve large amounts of complementarity with other products. For example, the value of owning a high definition television is dependent upon the amount of high definition programming that is available. Similarly, the value of owning a fuel cell car is dependent upon the availability of hydrogen fuel. There are also many examples in information technology, including the strong complementarity between video game stations, e.g. Sony Playstation, Microsoft X-box, etc. and gaming software. In many of these examples, the firms that produce one of the complementary products are ill-equipped to produce the other. Indeed, consumer electronics firms that produce high definition televisions, or video game stations typically do not produce programming or software. Nor would an automobile manufacturer be well-prepared to distribute hydrogen fuel, which is notoriously difficult to handle and store.

When such complementarities exist between the products of different firms, each firm has an incentive to under-produce because of the externality that causes it to ignore the positive effect of its own output upon the profits of the other firm. Complementarity creates a sort of prisoner's dilemma in which both firms would benefit if both firms increased their output, but neither firm has an incentive to unilaterally increase its own output.

We propose that, a firm that has patent protection for its technology can use an appropriately designed licensing arrangement to provide a credible commitment to a higher level of output than it would produce on its own, thereby encouraging output of the complement. When complementary effects are sufficiently strong, such a licensing arrangement can allow the firm to earn larger profits than it could by serving the market directly as a monopolist, even if the licensees have no production cost advantage. Note that this contrasts with the typical result for firms that have exclusive rights to product technologies that do not interact with complements.

The remainder of our paper is organized as follows: After reviewing the related literature, we introduce a model that captures interactions between producers of two complements. In section 3.3, we discuss the signaling strategy of the monopolists when the product is of standard nature. Finally, we summarize the paper and propose future research in section 4.6.

3.2 Related Literature

Complementary effects have long been a subject of interest in economics literature. Katz & Shapiro (1985*a*), Katz & Shapiro (1994); Farrell & Saloner (1985), (1986); Choi (1994) and Liebowitz & Margolis (1994) discuss the innovation, competition, and compatibility issues in the context of the complementary effects. However, they usually take a pair of complements as a system and ignore the possibilities of intended strategic signalling effects between two complementary industries.

Recent work by Parker & VanAlstyne (2003a) and Parker & VanAlstyne (2003b) examines how to stimulate demand by subsidizing one of the complementary markets. Our paper also studies how the complementary interactions between two markets can be used to influence demand, but we differ from theirs in that we assume that two complements are not produced by the same firm, which is commonplace in the real world. Bhaskaran & Gilbert (Forthcoming) demonstrate that selling may be better than renting for a durable good monopolist as selling can serve as a commitment to a higher future output level and thus the complementary producer would increase the output accordingly. We focus on different signaling mechanisms other than renting.

Our work is also related to the literature on the role of intermediaries. In their seminar paper, McGuire & Staelin (1983) demonstrate that, by selling through intermediaries, competing manufacturers can dampen the effects of competition. If their products are sufficiently substitutable, then they can earn higher profits selling through intermediaries than they could selling directly. Other papers, including Choi (1991), Gupta & Loulou (1998), etc. extended this line of analysis. But most of this work focused on how intermediaries affect competition between manufacturers and assumed that linear wholesale pricing is used. This contrasts with our work, where we examine the role that intermediaries can play in improving coordination between complementors and allow for non-linear wholesale pricing.

A lot of research has been done on capitalization of patented technology through licensing. Arrow (1962) compared the profit an inventor could realize by licensing it to a perfectly competitive industry versus a monopoly, by means of a per-royalty. Kamien & Schwartz (1982) studied the option of licensing to a oligopolistic industry by means of both a fixed fee and a royalty. Kamien & Tauman (1984) extended the work and analyzed how much profit an inventor can obtain by using both a fixed fee and a royalty in a purely competitive industry. Katz & Shapiro (1984) and Katz & Shapiro (1985*b*) have studied licensing by means of auction. Fershtman & Kamien (1992) showed that anticipation of crossing licensing of two complementary technology tends to retard each firm's development process. Shapiro (1985) and Kamien (1992) provide overviews of related work. All this research overlooked the possible interactions between patented technology and its complementary products, which is the focus of this paper.

3.3 The Model

We consider two complementary products, A and B, that are produced by two different firms. We assume that product A is durable in the sense that each consumer purchases at most one unit of it, while product B is non-durable, i.e. consumers may purchase multiple units of product B. These assumptions are consistent with the relationship that exists between a video-game station and video games, a high-definition television and high-definition programming, a fuel-cell car and hydrogen fuel, and many other complementary products. We consider only a single period, so we do not consider the time inconsistency issue that often exists in durable goods contexts.

In most of the examples given above, it is the producer of the durable product that has the most market power. Often, many different firms compete to produce the non-durable complement. For this reason, we adopt the perspective of the producer of product A, which we will refer to as "firm A", in our analysis. For simplicity, we assume that product B is produced by a single firm, "firm B". However, most of our results can also be obtained for situations in which we allow for free entry into the market for product B. We assume that both firms have constant marginal costs and normalize them to zero.

To capture the complementary interactions between the two products, we use the following adaptation of the model developed by Bhaskaran & Gilbert (Forthcoming). There are a total of M consumers, who will buy one unit of A or none. In the absence of product B, a consumer's utility for product A is v_A , which is uniform over $[a_A - M, a_A]$, where $0 \leq a_A \leq M$. The assumption that some consumers have negative utility for product A implies that there will always be some consumers who do not purchase the product at any price. Let δ be an indicator function that is equal to one if a consumer purchases product A, and is equal to zero otherwise.

For product B, we assume that each consumer has a decreasing marginal utility, and that this marginal utility is higher if he has the use of product A than if he does not. Specifically, each consumer has the following marginal utility for the y^{th} unit of product B: $(a_B + \delta k + \phi v_A - y)/\gamma$, where a_B, k, $\gamma \geq 0$ and $\phi \in (0, 2\gamma)$ are constants. The parameter a_B indicates the magnitude of consumers' utility for product B relative to product A; k represents the strength of complementarity; γ is a measure of price sensitivity; and ϕ indicates the relationship between a consumer's valuation for product A and his marginal utility for product B. If ϕ is strictly positive, then consumers with the highest valuations for the use of A will have the highest marginal utilities for product B. If $\phi = 0$, then all consumers are homogeneous with respect to their marginal utilities for the complement. A consumer's total utility can be expressed as the following function of whether he purchases product A, $\delta \in \{0, 1\}$, and the amount, y_i, of product B that he purchases:

$$U_i(y_i, \delta) = \delta\,(v_A - p_A) + \int_0^{y_i} \frac{a_B + k\delta + \phi v_A - x}{\gamma} dx - y_i p_B \tag{3.1}$$

where p_A, p_B are prices of product A and B. Note that by setting $k = 0$, we get the consumer's utility function when A and B are not complements. A larger k corresponds to a stronger complementary interaction. As described in Bhaskaran & Gilbert (Forthcoming), it can be shown that at price p_B, a utility maximizing individual consumer with valuation v_A for product A would consume

$$y_i(p_B, v_A, \delta) = a_A + k\delta + \phi v_A - \gamma p_B \tag{3.2}$$

units of product B. Thus, at a given price p_B, access to product A increases the amount of product B that a consumer will purchase by k units.

To determine how product B affects a consumer's willingness to pay for product A, we must consider his total utility as a function of the price of product B. If the price of B is p_B, and the consumer's independent valuation for product A is v_A, then having product A increases his total utility by the following amount:

$$\begin{aligned} U\left(y(p_B, v_A, 1), 1\right) \quad - \quad & U\left(y(p_B, v_A, 0), 0\right) = \\ & v_A - p_A + \frac{(a_B + k + \phi v_A - \gamma p_B)^2}{2\gamma} - \frac{(a_B + \phi v_A - \gamma p_B)^2}{2\gamma} \\ = \quad & v_A - p_A + \frac{k^2 + 2k(a_B + \phi v_A - \gamma p_B)}{2\gamma} \end{aligned} \tag{3.3}$$

Note that the latter term in this expression represents the amount by which the availability of product B increases a consumer's willingness to pay for product A. Thus, if the price charged for product A is p_A, then all consumers with an independent valuation of more than $p_A - \frac{k^2+2k(a_B+\phi p_A-\gamma p_B)}{2\gamma}$ will purchase product A, and the total number of consumers who pay to use the service of product A will be:

$$Q \;=\; \left[Min\left\{M,\, a_A + \frac{k^2 + 2k(a_B + \phi p_A - \gamma p_B)}{2\gamma} - p_A\right\}\right]^+ \tag{3.4}$$

To facilitate the analysis, we will introduce several restrictions upon our parameters:

$$a_B \leq \frac{2\gamma\,(M - a_A) - k^2}{2k} \tag{3.5}$$

$$a_B \geq \frac{k^3 + 2M\phi(\gamma + k\phi)(3M - 2a_A) + k^2\phi(5M - 2a_A) + 2k\gamma a_A}{2(k^2 + 2M\gamma + 2kM\phi)} \tag{3.6}$$

The first of these restrictions is sufficient to guarantee that at equilibrium, $Q \leq M$, i.e. some consumers will not purchase product A. This plays a major role in our results since it implies that a decrease in the price of product A leads to more consumers having access to it, which increases demand for product B. Note that because we have assumed that $a_B \geq 0$, assumption (3.5) also implies that $k^2 \leq 2\gamma(M - a_d)$. The restriction shown in (3.6) implies that, at equilibrium, all consumers purchase a positive amount of product B, even those who lack access to product A. Although this restriction simplifies the mathematical analysis, our results do not depend upon it qualitatively. Thus, even though (3.6) precludes the case in which $a_B = 0$, by sacrificing some clarity of exposition, our results can be extended to include this case as well. Note that the assumption that $\phi \geq 0$ implies that the right-hand-side of (3.5) is larger than the right-hand-side of (3.6).

From (3.2) and (3.4), we can obtain the following inverse demand functions for product A and product B:

$$p_A(Q, y) = \frac{(k^2 + Mk\phi)(M - 2Q) + 2M(a_A - Q)\gamma + 2ky}{2M\gamma} \quad (3.7)$$

$$p_B(Q, y) = \frac{2Ma_B + 2kQ - 2y + M\phi(2a_A - M)}{2M\gamma} \quad (3.8)$$

where Q and y are the numbers of units of product A and B that are available for consumers.

As should be expected for complementary products, the inverse demand function for each of products A and B is decreasing in its own quantity and increasing in the quantity of the other. As a result, firms A and B will have incentives to set quantities too low or prices too high. Bhaskaran & Gilbert (Forthcoming) use a two-period version of the above model in which period 1 output of product A competes with period 2 output in a secondary market. In this context, they show that when complementarity is high, a durable goods manufacturer (firm A) should sell his product instead of leasing it, as would typically be optimal for a durable goods manufacturer whose product does not interact with a complement. We will argue that an additional step that a durable goods manufacturer can take is to license its technologies to intermediaries, instead of selling directly to consumers. Note that in our model, we require only that each consumer purchase no more than one unit of product A; we do not consider a multi-period context in which today's output of product A competes with tomorrow's.

3.4 Benchmark Profits

To define a benchmark, let us first consider what would happen if firm A did not license the proprietary technology, and instead brought product A to market herself.

We define the profit that firm A earns by producing and selling the product directly by herself as her *proprietary direct (PD)* profit. To obtain our benchmark monopoly profit, we define the profit functions of firm A and B assuming that they set the quantities simultaneously and sell products directly to the market.

$$\pi_A^{PD}(Q,y) = Q\,p_A(Q,y) \tag{3.9}$$

$$\pi_B^{PD}(Q,y) = y\,p_B(Q,y) \tag{3.10}$$

where $p_A(Q,y)$ and $p_B(Q,y)$ are from equations (3.7) and (3.8).

In equilibrium, each firm determines its output in order to maximize its own profits. These equilibrium output quantities can be identified by simultaneously solving the first-order conditions for (3.9) and (3.10) with respect to Q and y respectively.

$$Q^{PD*} = \frac{M(2k(a_B+k)+kM\phi+2(2\gamma+k\phi)a_A)}{2(3k^2+4M\gamma+4kM\phi)}$$

$$y^{PD*} = \frac{M(k^3-M\phi k^2-2M^2\phi(\gamma+k\phi)+(4a_B+4\phi a_A)\,(k^2+M\gamma+kM\phi)+2a_Ak\gamma)}{2(3k^2+4M\gamma+4kM\phi)}$$

Substituting the resulting equilibrium output quantities back into (3.9) and (3.10), we have:

$$\pi_A^{PD*} = \frac{M(k^2 + M\gamma + kM\phi)(2k(a_B + k) + kM\phi + 2(2\gamma + k\phi)a_A)^2}{4\gamma(3k^2 + 4M\gamma + 4kM\phi)^2}$$

$$\pi_B^{PD*} = \frac{M(k^3 - M\phi k^2 - 2M^2\phi(\gamma + k\phi) + (4a_B + 4\phi a_A)(k^2 + M\gamma + kM\phi) + 2a_A k\gamma)^2}{4\gamma(3k^2 + 4M\gamma + 4kM\phi)^2}$$

3.5 Technology Licensing

One way in which a firm can capitalize on a proprietary technology that interacts with a complement is to license the technology to other firms. In this section, we explore two different forms of technology licensing: fixed fee, and royalty. Under a fixed fee arrangement, a licensee pays a one time fee for the right to produce the technology that is independent of the amount produced. Under a royalty arrangement, the licensee pays a royalty to the owner of the technology for each unit that is sold to a consumer. Note that under a royalty arrangement, the royalty paid by the licensee is analogous to the wholesale price that is paid by an intermediary to a supplier under a linear pricing scheme. Thus, our analysis of the royalty arrangement can also be applied to situations in which firm A sells its technology through intermediaries using a linear pricing rule.

3.5.1 Fixed-Fee License

To consider the fixed-fee licensing arrangement, we assume that there are a large number of potential licensees that will participate as long as they can earn non-negative profits. Of course, our analysis can easily be extended to require positive participation profits for the licensees. The product sold by all of the licensees is undifferentiated, and they have no production cost advantage or disadvantage relative to firm A. Recall that firm A's production cost has been normalized to zero.

Under a fixed-fee license arrangement, Firm A moves first by setting the one-time license fee, denoted by F, that a licensee must pay in order to participate in selling product A. In response to this fee, licensees enter the industry until each one earns his indifference profits, which we have assumed to be zero. Let n denote the number of licensees that pay the fixed fee of F in order to participate. For analytical tractability, we treat n as a continuous variable throughout the paper.

We assume that the license fee F is fully observable, so that firm B can anticipate the number of licensees. Therefore, following the licensees' entry, firm B and the licensees all make their output decisions simultaneously. Let q_{Ai} denote the output for product A from licensee $i = 1, ..., n$, and let $Q_{AT} = q_{A1} + ... + q_{An}$. Recall that y represents the output of firm B. In the output setting stage of the game, each of the n licensees determines the level of output that will maximize its profits, which can be represented as follows:

$$\pi_{Ai}^{FF}(q_{A1}, q_{A2}, ..., q_{An}, y, F) = q_{Ai} p_A(Q_{AT}, y) - F \tag{3.11}$$

for $i = 1, ..., n$, while firm B sets its output to maximize:

$$\pi_{B}^{FF}(q_{A1}, q_{A2}, ..., q_{An}, y, F) = y\, p_B(Q_{AT}, y) \tag{3.12}$$

By applying first-order conditions to (3.11) and (3.12), we can determine the output quantities of product A for any given number, n, of licensees. Denote these quantities as $q_{Ai}^*(n)$, $i = 1, ..., n$, and $y^*(n)$ respectively. However, because licensees enter until they anticipate that they cannot earn positive profits, in equilibrium we will also need to have:

$$\pi_{Ai}^{FF}(q_{A1}^*(n), q_{A2}^*(n), ..., q_{An}^*(n), y^*(n), F) = 0 \tag{3.13}$$

By simultaneously, solving (3.13) along with the first order conditions for (3.11) and (3.12), we can obtain the following expression for the equilibrium number of licensees that will participate in a fixed fee licensing arrangement when the fixed fee is set to F:

$$n^{FF}(F) \tag{3.14}$$
$$= \frac{\sqrt{k^2M + M^2\gamma + kM^2\phi}(k(2a_B + 2k + M\phi) + 2a_A(2\gamma + k\phi)) - 4\sqrt{F\gamma}(k^2 + M\gamma + kM\phi)}{2\sqrt{F\gamma}(k^2 + 2M\gamma + 2kM\phi)}$$

Thus, when firm A sets the licensing fee, it does so to maximize the following profit:

$$\pi_A^{FF}(q_{A1}^*(n), q_{A2}^*(n), ..., q_{An}^*(n), y^*(n), F) = Fn^{FF}(F) \tag{3.15}$$

It is easy to confirm that firm A's profits, as represented in (3.15) , are maximized when the fixed fee is set as follows

$$F^* = \frac{M(2k(a_B + k) + kM\phi + 2(2\gamma + k\phi)a_A)^2}{64\gamma(k^2 + M\gamma + kM\phi)}$$

In response to this fixed fee, the number of licensees and profits for firm A are the following:

$$n^{FF*} = \frac{2(k^2 + M\gamma + kM\phi)}{k^2 + 2M\gamma + 2kM\phi} \tag{3.16}$$

$$\pi_A^{FF*} = \frac{M(2k(a_B + k) + kM\phi + 2(2\gamma + k\phi)a_A)^2}{32\gamma(k^2 + 2M\gamma + 2kM\phi)} \tag{3.17}$$

Proposition 6 *Under a fixed fee licensing arrangement, the equilibrium number of licensees has the following properties:*

i) $n^{FF*} = 1$ *when* $k = 0$.

ii) n^{FF*} *is increasing in* k.

iii) $\lim_{\to \infty} n^{FF*} = 2$.

This result has several important implications. First, part *i)* confirms that, in the absence of complementary effects, the best that firm A can do is to set the fixed fee in such a way that when exactly one licensee participates, it earns zero profits. Since firm A is able to extract all of the profits from this licensee, it earns exactly the same profit that it would by selling its product directly to the market. Part *ii)* of the proposition confirms that as complementarity increases, firm A will induce more licensees to participate in the fixed fee arrangement. However, part *iii)* shows that firm A will never induce more than two licensees to enter. Recall that the equilibrium described in (3.16) and (3.17) is an approximation to the true equilibrium in which there must be an integer number of licensees. Therefore (3.17) represents an upper bound on the profit that firm A can earn under a fixed fee licensing arrangement.

Proposition 7 *There exists a threshold level of complementarity,* K*, such that if* $k^2 > K$*, then firm A can maximize its profits under a fixed fee licensing by inducing exactly two licensees to participate, and these profits will be larger than firm A could earn by selling its product directly to the market. This threshold value of* $K = (M\phi)^2 + \sqrt{2}M\gamma + M\phi\sqrt{M(2\sqrt{2}\gamma + M\phi^2)}$.

By inverting n^{FF*} as shown in (3.14) we can see that the fixed fee that is necessary to induce exactly $n = 2$ licensees to participate is equal to:

$$F^{FF}(2) = \frac{M(k^2 + M\gamma + kM\phi)(2k(a_B + k) + kM\phi + 2(2\gamma + k\phi)a_A)^2}{8\gamma(2k^2 + 3M\gamma + 3kM\phi)^2}$$

and the total profit earned by firm A from licensing its product to two licensees at this fixed fee is:

$$\pi_A^{FF}(2) = \frac{M(k^2 + M\gamma + kM\phi)(2k(a_B + k) + kM\phi + 2(2\gamma + k\phi)a_A)^2}{4\gamma(2k^2 + 3M\gamma + 3kM\phi)^2}$$

3.5.2 Royalty License

A common alternative to fixed fee licensing arrangements is a royalty fee arrangement. Under a royalty arrangement, firm A announces a per-unit fee, denoted by L, that is available to any potential licensee. Note that this arrangement is similar to one in which firm A sells its product through intermediaries at a pre-arranged wholesale price of L. However, since licensing arrangements tend to be more formal, they tend to allow less flexibility for firm A to adjust its price and are therefore more credible as mechanisms for strategic commitment.

As before, we assume that, following the announcement of the royalty fee, L, potential licensees enter as long as they do not earn negative profits. Again, we denote the number of licensees by n. Finally, firm B and the licensees simultaneously determine their quantities of output, denoted by q_B and q_{Ai}, for $i = 1, ..., n$ respectively.

Recall that, for fixed license agreements, the fixed fee has only an indirect effect upon the output decision of an individual licensee, through the number of of licensees that agree to participate. In contrast, a royalty fee has a direct impact on a licensee's output decision. Under a royalty arrangement, at the final stage of the game where quantities of output are being determined, the profit function for each licensee is as follows:

$$\pi^R_{Ai}(q_{A1}, q_{A2}, ..., q_{An}, y, L) = q_{Ai}(p_A(Q_{AT}, y) - L) \tag{3.18}$$

for $i = 1, ..., n$ while the profit for firm B and firm A are:

$$\pi^R_{B}(q_{A1}, q_{A2}, ..., q_{An}, y, L) = y\, p_B(Q_{AT}, y) \tag{3.19}$$

$$\pi^R_{A}(q_{A1}, q_{A2}, ..., q_{An}, y, L) = LQ_{AT} \tag{3.20}$$

By simultaneously solving the first-order conditions for (3.18) with respect to q_{Ai}, and (3.19) with respect to q_B, we can identify the equilibrium output quantities conditional upon the number n of licensees.

$$q^{R*}_{Ai}(n) = \frac{M(2k(a_B + k) - 4L\gamma + kM\phi + 2a_A(2\gamma + k\phi))}{2(k^2(2 + n) + 2M\gamma(1 + n) + 2kM\phi(1 + n))}$$

$$y^{R*}(n) = \frac{M(k^3 n - 2kn\gamma(L - 1) - M\phi(k^2 + G(1 + n)) + (2a_B + 2a_A\phi)(1 + n)(k^2 + G))}{2(k^2(2 + n) + 2M\gamma(1 + n) + 2kM\phi(1 + n))}$$

where $G = M\gamma + kM\phi$. However, it can be observed that, for any finite value of n each of the licensees earns a positive profit.

Proposition 8 *Under a royalty fee licensing arrangement, the optimal royalty fee, L^{R*} is independent of the number n of licensees, where:*

$$L^{R*} = \frac{2k(a_B+k)+kM\phi+2(2\gamma+k\phi)a_A}{8\gamma}$$

The profits of firm A are increasing in n and approach the following limiting value as $n \to \infty$:

$$\pi_A^{R*} = \pi_A^{FF*} = \frac{M(2k(a_B+k)kM\phi+2(2\gamma+k\phi)a_A)^2}{32\gamma(k^2+2M\gamma+2kM\phi)}$$

It is easy to confirm that the optimal royalty rate, L^{R*}, is strictly increasing with the complementary effect parameter, k. For a given number, n, of licensees, the profits of firm A, firm B, and each licensee are increasing in complementarity, k. If the number of potential licensees is large, and their participation profits / fixed costs are truly negligible, then a royalty licensing arrangement could potentially deliver a larger total profit to firm A than could a fixed fee arrangement. Recall that when complementary effects are strong, i.e., $k^2 > K$, firm A can earn larger profits by using a fixed fee licensing arrangement with two licensees than by selling directly to the market. On the other hand, if $k^2 < K$, then having one licensee is the optimal solution under a fixed-fee license, and firm A earns the same profit as it would by selling its product directly. The following proposition identifies the conditions under which a royalty licensing arrangement is preferable to a fixed-fee arrangement.

Proposition 9 *If a sufficiently large number of potential licensees will participate in a royalty arrangement, then firm A can earn larger profits from an optimal royalty arrangement than it can through either a fixed fee arrangement or from selling its product directly. The critical number of licensees depends upon k. If $k^2 < K$, then the minimum number of licensees needed for firm A's optimal profits under a royalty arrangement to dominate its profits under fixed fee is:*

$$n_1 = \frac{16}{k^4}\left((k^2 + M\gamma)^2 + \phi k(2Mk^2 + 2M^2\gamma + M^2\phi k)\right)$$

Otherwise, if $k^2 \geq K$ then the minimum number of licensees needed for firm A's optimal profits under a royalty arrangement to dominate its profits under fixed fee *is:*

$$n_2 = \frac{8\Big((k^2+M\gamma)^2+\phi k(2Mk^2+2M^2\gamma+M^2\phi k)\Big)}{M^2(\gamma+k\phi)^2}$$

Recall from Proposition 7 that $K = (M\phi)^2+\sqrt{2}M\gamma+M\phi\sqrt{M(2\sqrt{2}\gamma+M\phi^2)}$. Note that when there is no complementary interaction, i.e. $k = 0$, the threshold number of licensees, n_1 becomes infinite. In this case, double marginalization prevents firm A from earning as much by selling through intermediaries as it could by selling its product directly or through a single licensee from whom it could extract all of the profits through a fixed licensing fee. For relatively low values of complementarity, i.e. $k^2 < K$, firm A's optimal fixed-fee licensing arrangement induces only one licensee to pay the fixed-fee and results in the same profits that firm A would earn by selling her product directly. However, even for these low values of complementarity, if a sufficiently large number of licensees will participate in a royalty arrangement, then firm A can use royalty licensing to earn greater profits than she could by either selling her product directly or by selling through fixed fee licencing. For larger values of complementarity, $k^2 \geq K$, firm A can also earn more through royalty licensing as long as the barriers to entry for licensees are low enough to allow a critical number of them to enter.

3.5.3 Hybrid License

After analyzing both a pure fixed-fee license and a pure royalty license, we will now consider a hybrid form of licensing arrangement that includes both a fixed-fee and a per-unit fee. As before, we assume that firm A moves first to announce both a one-time, fixed licensing fee (F) paid by each licensee and a royalty fee (L) that is assessed on each unit that a licensee sells. Potential licensees respond to this announcement by entering until it is anticipated that further entry would result in negative profits (or profits that are below some minimum participation level). Finally, following the entry of the licensees, firm B and the licensees determine their output quantities simultaneously. As before, q_{Ai} denotes the output of licensee $i = 1, ..., n$, Q_{AT} denotes the combined output of all of the licensees, and y denotes the output of firm B. At the final stage of the game, where quantities of output are being determined, the profit function for each licensee is as follows:

$$\pi^R_{Ai}(q_{A1}, q_{A2}, ..., q_{An}, y, L) = q_{Ai}(p_A(Q_{AT}, y) - L) - F \tag{3.21}$$

for $i = 1, ..., n$ while the profit functions for firm B and firm A are:

$$\pi^R_B(q_{A1}, q_{A2}, ..., q_{An}, y, L) = y\, p_B(Q_{AT}, y) \tag{3.22}$$

$$\pi^R_A(q_{A1}, q_{A2}, ..., q_{An}, y, L) = nF + LQ_{AT} \tag{3.23}$$

By simultaneously solving the first-order conditions for (3.21) with respect to q_i, for $i = 1, ..., n$, and (3.22) with respect to y, we can identify the following output quantities for a given number (n) of licensees and royalty payment (L).

$$q^H_{Ai}(n, L) = \frac{M(2k(a_B + k) - 4L\gamma + kM\phi + 2a_A(2\gamma + k\phi))}{2(k^2(2+n) + 2M\gamma(1+n) + 2kM\phi(1+n))} \tag{3.24}$$

$$y^H(n, L) \tag{3.25}$$
$$= \frac{M(k^3 n - 2kn\gamma(L-1) - M\phi(k^2 + G(1+n)) + (2a_B + 2a_A\phi)(1+n)(k^2+G))}{2(k^2(2+n) + 2M\gamma(1+n) + 2kM\phi(1+n))}$$

where $G = M\gamma + kM\phi$. Note that neither of these quantities depends upon the fixed-fee, F. Recall that at the entry stage of the game, licensees enter until they earn zero profits. To determine the magnitude of the fixed-fee, that together with the royalty fee of L would induce exactly n licensees to enter, we substitute (3.24) and (3.25) into (3.21) and solve for the fixed fee that gives each licensee zero profit. By doing this, we obtain the following expression for the fixed-fee that will induce the entry of n licensees when the royalty rate is L:

$$F^H(n, L) = \frac{M(k^2 + M\gamma + kM\phi)(2k(a_B + k) - 4L\gamma + kM\phi + 2a_A(2\gamma + k\phi))^2}{4\gamma(k^2(2 + n) + 2M\gamma(1 + n) + 2kM\phi(1 + n))^2}$$

By substituting this function into (3.23), firm A's profits can be represented as a function of n and L. Note that even though firm A's direct decisions are F and L, she is implicitly determining the value of n when she designs the licensing arrangement. We have simply introduced a change of variables to facilitate the analysis.

From the first order conditions for firm A's profit, as a function of n and L, it can be confirmed that the hybrid licensing arrangement that maximizes the profits of firm A satisfies the following:

$$L^* = \frac{(k^2(n - 2) + 2M\gamma(n - 1) + 2kM\phi(n - 1))(2k(a_B + k) + kM\phi + 2(2\gamma + k\phi)a_A)}{8n\gamma(k^2 + 2M\gamma + 2kM\phi)}$$

In addition, firm A's profits are increasing in F, so for any number of licensees, she sets the fixed fee just high enough to allow them to make zero profits:

$$F^* = \frac{M(k^2 + M\gamma + kM\phi)(2k(a_B + k) + kM\phi + 2(2\gamma + k\phi)a_A)^2}{16n^2\gamma(k^2 + 2M\gamma + 2kM\phi)^2}$$

Proposition 10 *Under a hybrid licensing arrangement, firm A's optimal profits are equal to π_A^{FF*} and can be obtained for any number, n, of licensees.*

The optimal royalty license fee is concave increasing in the number of licensees (n) and bounded above by $\frac{2k(a_B+k)+kM\phi+2(2\gamma+k\phi)a_A}{8\gamma}$. As firm A decreases the fixed-fee to induce more licensees to enter, it will also increase the royalty rate.

The royalty fee L plays an important role in the effort to trade off between the complementary effects and the competition. Firm A uses L to induce the right quantity from each licensee such that the total output level is above the monopoly output yet not as high as the output that would be produced by two or more licensees in a pure fixed-fee arrangement. After observing the royalty rate of the licensing contract, firm B anticipates that a larger quantity of A will be produced and thus increases his output level correspondingly. Our result shows that, for any number of licensees, with the help of fixed fee to squeeze the licensees, firm A can use an appropriate royalty rate L to perfectly balance the complementary effects and competition.

The case when $n = 1$ is of special interest. From the expression of L^*, it is clear that when $n = 1$, $L^* < 0$ and when $n > 1$, $L^* > 0$. Recall that under a pure fixed-fee arrangement, firm A earns exactly the same profits from using a single licensee as it does by selling directly. Under a pure royalty arrangement, double marginalization prevents firm A from earning as much by licensing to a single licensee as it would be selling directly. However, under a hybrid arrangement, when $n = 1$, the variable licensing fee is negative, which means firm A subsidizes the only licensee at a per unit basis but charges a positive fixed fee. The purpose of negative royalty rate is to assure to firm B that the only licensee will produce more than monopoly output. This raises an interesting question: Why firm A does not produce more to earn more profit? This is because for firm A to produce more than the monopoly output is not the best response to producer B's decisions, thus producer B would not increase the quantity. The licensing arrangement here becomes a credible commitment to the future output. By licensing, firm A exploits the complementary effects and encourages higher output of B. However, it is possible that this single licensee has incentives to inflate the actual sales in order to obtain extra subsidy from firm A. Thus, the optimal hybrid licensing arrangement with a single licensee should be implemented only when actual sales can be easily monitored. Otherwise, firm A should seek entering the hybrid licensing arrangement with at least two licensees.

If firm A could squeeze every penny out of the licensees, a licensing structure as proposed above would lead to the maximum possible profit for firm A. In reality, firm A may have to give up some profit margin to the licensees. If such maximum possible licensing proceeds are high enough, firm A still has a chance to earn a profit better than monopoly profit after sharing profits with licensees.

Note that the hybrid licensing will lead to the exact monopoly profit for firm A if her product is complementary effect free. So for firms who are not indirectly interacting with a related industry, they may choose to produce and sell the product directly to the end market instead of dealing with the licensing contracts.

3.6 Concluding Remarks

In this paper, we show that a monopolist operating under the complementary effects behaves differently from a monopolist who is independent of other markets. These differences stem from the nature of the market dynamics that call for coordination across industries.

In the context of complementary interactions, licensing arrangements may deliver a profit that is better than selling the product directly to the market. Inserting intermediaries into the supply chain associated with right regulatory contracts can best balance the complementary effects and competition. The competition among licensees leads to higher output levels, which could encourage the complementary producer to produce more and stimulate the demand in both markets. On the other hand, the competition may significantly reduce the total profit that all licensees can earn and thus the licensing proceeds a technology patent owner can collect through licensing. An appropriate licensing arrangement can best trade off between complementarity and competition and bring better profit to the inventor of the technology. We reveal that the optimal output for a product interacting with a complementary industry is higher than monopoly output but less than the quantity would be produced by two licensees under fixed-fee licensing arrangement.

Three different licensing arrangements are considered in this paper. First of all, if the complementary effects is strong, we can utilize a fixed fee licensing contract with two licensees. However, we show that if there are enough potential licensees, then royalty licensing is always better than a fixed fee arrangement or a strategy to sell directly to the market. Finally, if both a fixed fee and a royalty can be used, then the optimal profit for the technology patent owner can be achieved under any number of licensees. The royalty fee induces the optimal output level from each licensee and cushions the competition among them. Fixed fee is used to mitigate the double-marginalization in such a technology licensing relationship.

Finally, the optimal licensing arrangement can be implemented internally. For instance, the technology patent owner can set up subordinate facilities and impose fixed fee and per unit transferring price. However, often the internal arrangement is not as transparent as an open licensing arrangement and thus lacks credibility to the complementary producer.

In this paper, we assume licensees do not have cost advantage. However, cost advantage of licensees is commonly assumed to be the reason for licensing. It may be of interest to study the optimal licensing structure when the technology patent owner and the potential licensees have different production costs. In addition, the technology patent owner may also compete with licensees by producing the same product and even moving before the licensees to set the quantity or price of the product. All this may be worth exploring in the future. Finally, empirical evidence on the signalling effect of licensing in the context of complementary interactions is also needed.

Chapter 4

Product Line Strategy under Complementary Effects

4.1 Introduction

For companies whose products interact with a complementary market, their financial performance greatly depends upon the dynamics in the related complementary industry. For example, for high definition TV to reach the mass market it is critical to have sufficient amount of high definition TV programming available at affordable prices. But companies that provide high definition broadcasting would increase the availability of high definition programming only when they expect high definition TV manufacturers to increase output. Thus, TV manufacturers may have incentives to assure high definition programming providers that there would be enough demand for their services.

Companies involved in complementary interactions usually produce less than the first best levels as they tend to ignore the fact that an increase in own output quantity has a positive effect on the demand for the other. If a firm interacting with a complementary producer can commit to a higher output level in advance, then the complementary producer may increase the quantity in response to such commitment, which in turn would benefit the firm as it makes the product more attractive with the use of the complement to a customer.

Multiple mechanisms can be used as commitments to higher output in order to coordinate the interests of companies interacting with each other through indirect complementary effects. For example, holding excess capacity or selling through licensees are all effective ways to convince complementary producers of higher future output. In this work we focus on the signalling effects of a firm's product line strategy and argue that providing a broad product line can constitute a credible promise to the complementary industry. In general, the purpose of providing vertically differentiated products to diversified consumers is to pursue higher profits through discrimination. However, our study shows that product line decisions are also strategically important in encouraging output from a complementary producer.

We study a firm's product line strategy under direct selling and technology licensing. Companies provide different product combinations when they deal with complementary effects of different strength. Their distribution decisions, i.e., whether to serve the market directly or to license the technology to other manufacturing firms, also have an effect on product line choices. We show that under complementary effects, a firm's product line strategy and its licensing arrangement are strategic complements.

The remainder of our paper is organized as follows: After reviewing the related literature, we introduce a model that captures interactions between producers of two complements and a firm's quality decisions. In section 4.4, the product line decision of a direct selling monopolist is studied. In section 4.5, the product line decision under technology licensing is studied and compared to the direct selling case. Finally, we summarize the paper and propose future research in section 4.6.

4.2 Related Literature

Complementary effects have long been a subject of interest in economics literature. Katz & Shapiro (1985*a*), Katz & Shapiro (1994); Farrell & Saloner (1985), (1986); Choi (1994) and Liebowitz & Margolis (1994) discuss the innovation, competition, and compatibility issues in the context of the complementary effects. However, they usually take a pair of complements as a system and ignore the possibilities of intended strategic signalling effects between two complementary industries.

Recent work by Parker & VanAlstyne (2003a) and Parker & VanAlstyne (2003b) examines how to stimulate demand by subsidizing one of the complementary markets. Our paper also studies how the complementary interactions between two markets can be used to influence demand, but we differ from theirs in that we assume that two complements are not produced by the same firm, which is commonplace in the real world. Bhaskaran & Gilbert (Forthcoming) demonstrate that selling may be better than renting for a durable good monopolist as selling can serve as a commitment to a higher future output level and thus the complementary producer would increase the output accordingly. We focus on different signaling mechanisms other than renting and we consider the firm's quality decisions as well.

Our paper studies the alternative role that product line decisions can play. We draw on the classic literature on market segmentation and product line strategies (Moorthy & Png (1992), Mussa & Rosen (1978)) and establish that a monopolist interacting with a complementary market may choose a broader product line than he would if the product he produces was independent of other markets. Our results also confirm a finding of Bhargava & Choudhary (2001) that a monopolist only provides a single version of the product if the product is independent of other markets and the production costs do not vary greatly with quality. We demonstrate that licensing would encourage the adoption of a broader product line, thus licensing and providing a broader product line are strategic complements. Bulow et al. (1985) were the first to establish the concept of strategic complements. They show that a firm's actions in one market can change competitors' strategies in a second market. We establish that a firm's decisions in one market can change its complementary producer's strategy.

Our work is also related to the literature on the role of intermediaries. In their seminal paper, McGuire & Staelin (1983) demonstrate that, by selling through intermediaries, competing manufacturers can dampen the effects of competition. If their products are sufficiently substitutable, they can earn higher profits selling through intermediaries than they could selling directly. Other papers, including Choi (1991), Gupta & Loulou (1998), etc. extended this line of analysis. But most of this work focused on how intermediaries affect competition between manufacturers and assumed that linear wholesale pricing is used. This contrasts with our work, where we examine the role that intermediaries can play in improving coordination between complementors and allow for non-linear wholesale pricing when there are multiple versions of the product are provided.

4.3 The Basic Model

We consider a durable good A and its non-durable complement B. These two products are produced by two different firms. Firm A can choose to offer two different versions of product A with quality levels $s_L < s_H = 1$ or provide a single version of the product to the market. It is assumed that there are M consumers who will buy one unit of A or none. In the absence of product B, a consumer's utility derived from consuming a unit of product A is $v_A s$, where s is the quality of the product and v_A is a random variable uniformly distributed over $[a_A - M, a_A]$, where $0 \leq a_A \leq M$.

Here v_A is a measure of consumers' utility per unit of quality; while $v_A s_L$ and $v_A s_H$ are consumers' reservation prices for low and high end versions of product A. The variable cost for producing a unit of A is c_A, for both high and low end versions. The constant marginal cost across quality levels is a plausible assumption if we consider the products that are digital in nature, such as application software. Our model differs from that of Bhaskaran & Gilbert (Forthcoming) in that we consider a firm's quality decisions and production costs.

For simplicity, product B is assumed to be non-differentiated and the variable production costs are normalized to zero. If a consumer does not have the use of product A, his marginal utility for the y^{th} unit of product B can be expressed as $(a_B + \phi v_A - y)/\gamma$, where a_B, $\gamma \geq 0$ and $\phi \in (0, 2\gamma)$ are constants. Note that if ϕ is strictly positive, then consumers with the highest valuations for the use of A will have the highest marginal utilities for product B. If $\phi = 0$, then all consumers are homogeneous with respect to their marginal utilities for the complement. If a consumer has the use of product A, either a high end version or a low end version, then his marginal utility for the y^{th} unit of product B increases by k/γ. That is, the marginal utility for the y^{th} unit of product B becomes $(a_B + k + \phi v_A - y)/\gamma$. Here we assume the same k for both high end version and low end version. This is plausible when having either a high end or a low end version of product A, a consumer's evaluation toward complementary product B is affected to the same extent.

We first derive the indifferent consumers. A consumer with valuation v_A will buy the following number of product B, given the price of product B (p_B).

$$y_i(p_B, v_A, \delta) = a_B + k\delta + \phi v_A - \gamma p_B \tag{4.1}$$

where δ is a {0,1} valued indicator. If $\delta = 1$, the consumer possesses a unit of either high end or low end version of product A. Otherwise, the consumer consumes only product B.

A consumer's total utility from consuming a unit of product A with high quality and $y_i(p_B, v_A, 1)$ units of B is

$$(v_A s_H - p_{AH}) + \int_0^{y_i(p_B, v_A, 1)} \frac{a_B + k + \phi v_A - x}{\gamma} dx - y_i(p_B, v_A, 1) p_B \tag{4.2}$$

where p_{AH} is the unit price for the high end version of product A.

The total utility from consuming a unit of A of low quality and $y_i(p_B, v_A, 1)$ units of B is

$$(v_A s_L - p_{AL}) + \int_0^{y_i(p_B, v_A, 1)} \frac{a_B + k + \phi v_A - x}{\gamma} dx - y_i(p_B, v_A, 1) p_B \tag{4.3}$$

where p_{AL} is the unit price for the low end version of product A.

By setting (4.2) and (4.3) equal, we can derive our marginal consumer who is indifferent between buying a unit of high end version or a unit of low end version of product A. Such marginal consumer's utility per unit of quality is v_A^1, which satisfies the following equation.

$$v_A^1 = \frac{p_{AH} - p_{AL}}{s_H - s_L}$$

The total utility for a consumer who buys only product B is

$$\int_0^{y_i(p_B, v_A, 0)} \frac{a_B + \phi v_A - x}{\gamma} dx - y_i(p_B, v_A, 0) p_B \tag{4.4}$$

By setting (4.4) equal to (4.3), we derive the second type of marginal consumer who is indifferent between buying a low end version of A and not buying. Her utility per unit of quality v_A^2 has to satisfy:

$$v_A^2 = \frac{2\gamma(kp_B + p_{AL}) - k(2a_B + k)}{2(s_L\gamma + k\phi)}$$

Based on the utilities of the marginal consumers, we can derive the inverse demand functions for both high and low end product A. Let Q_{AH} and Q_{AL} be the sales of high and low end version of product A, respectively. We assume that

$$a_A < \frac{4\gamma\left(c_A + 2Ms_L\right) + k\left(-2a_B + 2k + 7M\phi\right)}{4s_L\gamma + 2k\phi}$$

so that we have $Q_{AH} + Q_{AL} < M$.

The inverse demand functions for high and low end version of product A and for product B are:

$$\begin{aligned} & p_{AH}\left(Q_{AH}, Q_{AL}, y\right) \\ = & \frac{\left(k^2 + kM\phi\right)\left(M - 2\left(Q_{AH} + Q_{AL}\right)\right) + 2ky + 2M\gamma\left(a_A - Q_{AH} - Q_{AL}s_L\right)}{2M\gamma} \end{aligned} \tag{4.5}$$

$$\begin{aligned} & p_{AL}\left(Q_{AH}, Q_{AL}, y\right) \\ = & \frac{\left(k^2 + kM\phi\right)\left(M - 2\left(Q_{AH} + Q_{AL}\right)\right) + 2ky + 2M\gamma s_L\left(a_A - Q_{AH} - Q_{AL}\right)}{2M\gamma} \end{aligned} \tag{4.6}$$

$$p_B(Q_{AH}, Q_{AL}, y) = \frac{2Ma_B + 2k\left(Q_{AH} + Q_{AL}\right) - 2y + M\phi(2a_A - M)}{2M\gamma} \tag{4.7}$$

where y is the number of units of product B that are available for consumers.

4.4 The Product Line Decision under Direct Selling

We assume that firm A produces and sells one or two versions of product A directly to the market. If both versions are introduced, then more consumers will buy the product. However, the low end product will certainly cannibalize the high end market. So the product line decisions are made to best trade off between the increased volume and switching of high end consumers to low end product in a setting where product A is independent of any other markets. However, when product A and B are complements and produced by different companies, producer B's reaction to firm A's product line decisions should also be taken into consideration when firm A determines her product line.

4.4.1 When a Single Version Is Provided

If firm A only offers one version of the product, she would introduce the high end product as the production costs are the same for the high end and the low end products but consumers value the high end product more. The inverse demand functions have been derived in Chapter 3 and are shown below:

$$p_A(Q_H, y) = \frac{(k^2 + Mk\phi)(M - 2Q_H) + 2M(a_A - Q_H)\gamma + 2ky}{2M\gamma} \tag{4.8}$$

$$p_B(Q_H, y) = \frac{2Ma_B + 2kQ_H - 2y + M\phi(2a_A - M)}{2M\gamma} \tag{4.9}$$

Thus the profit functions of firm A and B can be expressed as

$$\pi_A^{SD}(Q_H, y) = Q_H\ (p_A(Q_H, y) - c_A) \quad (4.10)$$

$$\pi_B^{SD}(Q_H, y) = y\, p_B(Q_H, y) \quad (4.11)$$

where $p_A(Q_H, y)$ and $p_B(Q_H, y)$ are from equations (4.8) and (4.9).

In equilibrium, each firm determines its output to maximize its own profits. These equilibrium quantities can be identified by simultaneously solving the first order conditions for (4.10) and (4.11) with respect to Q_H and y respectively.

$$Q_H^* = \frac{M\,(2k\,(a_B + k) - 4\gamma c_A + kM\phi + 2a_A(2\gamma + k\phi))}{2\,(3k^2 + 4M\gamma + 4kM\phi)}$$

$$y^* = \frac{M}{2\,(3k^2 + 4M\gamma + 4kM\phi)}\,(\,k^3 + 2(a_A - c_A)k\gamma - M\phi\left(k^2 + 2M\gamma + 2kM\phi\right) + + (4a_B + 2\phi a_A)\left(k^2 + M\gamma + kM\phi\right)\,)$$

Substituting the resulting equilibrium output quantities back into (4.10) and (4.11), we have:

$$\pi_A^{SD*} = \frac{M\,(k^2 + M\gamma + kM\phi)\,(2k\,(a_B + k) - 4\gamma c_A + kM\phi + 2a_A(2\gamma + k\phi))^2}{4\gamma\,(3k^2 + 4M\gamma + 4kM\phi)^2} \quad (4.12)$$

$$\pi_B^{SD*} = \frac{M}{4\gamma\,(3k^2 + 4M\gamma + 4kM\phi)^2}\,(\,k^3 + 2(a_A - c_A)k\gamma - - M\phi\left(k^2 + 2M\gamma + 2kM\phi\right) + (4a_B + 2\phi a_A)\left(k^2 + M\gamma + kM\phi\right)\,)^2 \quad (4.13)$$

When $k = 0$, i.e., when there is no complementary interaction between firm A and B, firm A's optimal output and profits are:

$$Q_H^{SDI*} = \frac{a_A - c_A}{2}$$
$$\pi_A^{SDI*} = \frac{(a_A - c_A)^2}{4}$$

4.4.2 When Both Versions Are Provided

If firm A offers both high end and low end versions of product A, more consumers will be attracted to purchase a unit of A. The profit functions of firm A and firm B:

$$\pi_A^{BD}(Q_{AH}, Q_{AL}, y) = Q_{AH}(p_{AH}(Q_{AH}, Q_{AL}, y) - c_A) + Q_{AL}(p_{AL}(Q_{AH}, Q_{AL}, y) - c_A) \tag{4.14}$$

$$\pi_B^{BD}(Q_{AH}, Q_{AL}, y) = y p_B(Q_{AH}, Q_{AL}, y) \tag{4.15}$$

where $p_{AH}(Q_{AH}, Q_{AL}, y)$, $p_{AL}(Q_{AH}, Q_{AL}, y)$, and $p_B(Q_{AH}, Q_{AL}, y)$ are defined in equation (4.5), (4.6) and (4.7).

Firm A and firm B set their own output to maximize their respective profits. By applying first order conditions to (4.14) and (4.15), we can determine the equilibrium output quantities of product B and high end and low end of product A:

$$Q_{AH}^{BD*} = a_A/2 \tag{4.16}$$

$$Q_{AL}^{BD*} = \frac{M\left(2k\left(a_B+k\right)-4\gamma c_A+kM\phi\right)-a_A k(3k+2M\phi)}{2\left(3k^2+4Ms_L\gamma+4kM\phi\right)} \tag{4.17}$$

$$y^{BD*} = \frac{M}{2\left(3k^2+4Ms_L\gamma+4kM\phi\right)}\big(k^3+2k\gamma(a_A s_L-c_A)+k^2\phi(4a_A-M)+ 2M\phi(2a_A-M)(s_L\gamma+k\phi)+4a_B(k^2+Ms_L\gamma+kM\phi)\big) \tag{4.18}$$

Note that only when $M\left(2k\left(a_B+k\right)-4\gamma c_A+kM\phi\right)-a_A k(3k+2M\phi)>0$, firm A would provide the low end version in addition to the high end. Furthermore, $M\left(2k\left(a_B+k\right)-4\gamma c_A+kM\phi\right)-a_A k(3k+2M\phi)$ is continuous in k and negative when $k=0$. It equals zero at some $k_1<0$ and $k_2>0$. As k is positive by our assumption, it follows that only when $k>k_2>0$, firm A would offer both versions to the market. We denote this threshold of k beyond which firm A would provide both versions of product A if she sells directly to the market as K_D.

Note that it is straightforward to verify that when $k>K_D$, the total output of high and low end product is larger than the output when only the high end product is provided.

Substituting the resulting equilibrium output quantities back into (4.14) and (4.15), we have firms' equilibrium profits π_A^{BD*} and π_B^{BD*} when $k>K_D$.

Proposition 11 *When the complementary effects are strong, that is, when $k>K_D$, then a direct seller introduces a low end product.*

In order to prove above Proposition, we compare the equilibrium profits under a single product with equilibrium profits when there are two versions available when $k>K_D$. The Proposition shows that when the complementary effects are strong enough, firm A would offer both versions to the market. By introducing a low end version, the total number of customers who buy product A would increase and thus would encourage the manufacturer of product B to increase the output as well. As a result, customers may be willing to pay more for product A since product A and B are complements. If such complementary positive feedback loop is strong enough, firm A would benefit from introducing the low end.

Corollary 2 *When product A is independent of other product, i.e., $k = 0$, a direct seller only provides the high end version.*

The Corollary follows from the fact that when $k = 0$, the equilibrium low end quantity is negative. The Corollary confirms that firm A behaves differently when her product is independent of other markets and when she interacts with a complementary producer. Firm A under complementarity may provide a broader product line than she would if her product were independent of other markets. It also shows that the purpose of introducing the low end is to assure the complementary producer of higher future output levels. A broader product line attracts more consumers, which constitute a credible commitment to a higher output level. As producer B anticipates more product A would be sold, she will increase her output level as well, which in turn will increase the consumers' willingness to pay for product A. The result shows that the complementary effects can offset the cannibalization between high and low end product and make a broader product line an attractive option to a monopolist.

Corollary 2 also confirms Bhargava & Choudhary (2001)'s conclusion that if we can cost effectively produce the high end product relative to the low end and the distribution of the customer satisfies the Increasing Failure Rate property, then providing only the high end product will bring the maximum profits to the company.

Intuitively, as the high end product may have much higher profit margin due to constant variable production costs across qualities, the loss from cannibalization is large and thus introducing low end product is not attractive in the absence of complementary effects.

Corollary 3 *For a direct seller interacting with a complementary producer, the quality level of the low end has no effect on her decision regarding whether or not to include the low end into the product line.*

The Corollary 3 follows from the fact that the threshold K_D is the value of $k > 0$ that satisfies $2k\,(a_B + k) - 4\gamma c_A + kM\phi - a_A k(3k + 2M\phi) = 0$ and the equation does not contain s_L. From (4.17), although the low end output Q_{AL}^{BD*} decreases with s_L, but s_L does not affect the decision regarding whether or not to introduce the low end product.

Corollary 4 *When both versions would be available under direct selling ($k > K_D$), the lower is s_L, the better.*

Proof:

Take the first order derivative of π_A^{BD*} with respective to s_L. It can be shown that under the condition $k > K_D$, the derivative is negative. $\Diamond$

The Corollary shows that if both versions are provided, then firm A has an incentive to lower the quality of the low end version in order to get more consumers to buy.

4.5 Product Line Decision under Hybrid Technology Licensing

Instead of selling directly to the market, firm A could license the technology to other manufacturing firms. We assume a hybrid licensing arrangement is adopted. A royalty rate is charged based on the sales of licensees. Each licensee has to pay a one time fixed fee in order to participate in the licensing arrangement. We assume that potential licensees possess no cost advantage. All licensees incur a per unit production cost of c_A. Given the fixed fee and royalty rate of the license arrangement, potential licensees continue to enter the licensing relationship with firm A until it is not profitable to do so. Of course, our analysis can easily be extended to require positive participation profits for the licensees. We also assume that all licensees move simultaneously with producer B to set the output quantities to maximize their own profits.

4.5.1 When Only High End Version Is Offered

When firm A decides to provide only high end version to the market through licensing, the inverse demand functions for product A and for product B are as in equations (4.8) and (4.9). We assume that firm A moves first to announce both a one-time fixed licensing fee (F) paid by each licensee and a royalty fee (L) that is assessed on each unit that a licensee sells. Potential licensees respond to this announcement by entering until it is anticipated that further entry would result in negative profits (the results can be extended to case where each licensee incurs a fixed fee to enter or requires a certain level of return.) Finally, following the entry of the licensees, firm B and the licensees determine their output quantities simultaneously. Let q_{Ai} denote the output of licensee $i = 1, ..., n$ and Q_{AT} denotes the combined output of all of the licensees. y is still the output of firm B.

At the final stage of the game, where quantities of output are being determined, the profit function for each licensee is as follows:

$$\pi_{Ai}^{SL}(q_{A1}, q_{A2}, q_{A3}, ..., q_{An}, y, L, F) = q_{Ai}(p_A(Q_{AT}, y) - c_A - L) - F \tag{4.19}$$

for $i = 1, ..., n$ and $p_A(Q_{AT}, y)$ is from equation (4.8).

The profit functions for firm B and firm A are:

$$\pi_B^{SL}(q_{A1}, q_{A2}, ..., q_{An}, y, L, F) = y\, p_B(Q_{AT}, y) \tag{4.20}$$

$$\pi_A^{SL}(q_{A1}, q_{A2}, ..., q_{An}, y, L, F) = nF + LQ_{AT} \tag{4.21}$$

where $p_B(Q_{AT}, y)$ is from equation (4.9).

By simultaneously solving the first-order conditions for (4.19) with respect to q_{Ai}, for $i = 1, ..., n$, and (4.20) with respect to y, we can identify the following output quantities for a given number (n) of licensees and royalty payment (L).

$$q_{Ai}^{SL}(n, L) = \frac{M(2k(a_B + k) - 4\gamma(L + c_A) + kM\phi + 2a_A(2\gamma + k\phi))}{2(k^2(2 + n) + 2M\gamma(1 + n) + 2kM\phi(1 + n))} \tag{4.22}$$

$$y^{SL}(n, L) = \frac{M}{2(k^2(2 + n) + 2M\gamma(1 + n) + 2kM\phi(1 + n))} \big((k^3 n - 2kn\gamma(a_A - c_A - L) + $$
$$+ (1 + n)\big(2(a_A\phi + a_B)(k^2 + M\gamma + kM\phi) - M\phi(M\gamma + kM\phi)\big) - k^2 M\phi\,) \tag{4.23}$$

Note that neither of these quantities depends upon the fixed-fee, F. Recall that at the entry stage of the game, licensees enter until they earn zero profits. To determine the magnitude of the fixed-fee, that together with the royalty fee of L would induce exactly n licensees to enter, we substitute (4.22) and (4.23) into (4.19) and solve for the fixed fee that gives each licensee zero profit. By doing this, we obtain the following expression for the fixed-fee that will induce the entry of n licensees when the royalty rate is L:

$$F^{SL}(n, L) = \frac{M(k^2 + M\gamma + kM\phi)(2k(a_B + k) - 4\gamma(c_A + L) + kM\phi + 2a_A(2\gamma + k\phi))^2}{4\gamma(k^2(2+n) + 2M\gamma(1+n) + 2kM\phi(1+n))^2}$$

By substituting this function into (4.21), firm A's profits can be represented as a function of n and L. Note that even though firm A's direct decisions are F and L, she is implicitly determining the value of n when she designs the licensing arrangement. We have simply introduced a change of variables to facilitate the analysis.

From the first order conditions for firm A's profit, as a function of n and L, it can be confirmed that the hybrid licensing arrangement that maximizes the profits of firm A satisfies the following:

$$\begin{aligned} L^{SL*} = & \frac{(k^2(n-2) + 2M\gamma(n-1) + 2kM\phi(n-1))}{8n\gamma} \times \\ & \times \frac{(2k(a_B + k) - 4\gamma c_A + kM\phi + 2(2\gamma + k\phi)a_A)}{(k^2 + 2M\gamma + 2kM\phi)} \end{aligned}$$

The optimal royalty rate is concave increasing in the number of licensees (n) and bounded above. This is because when more licensees enter, firm A charges a higher royalty fee in order to keep the total output at the optimal level.

In addition, firm A's profits are increasing in F, so for any number of licensees, she sets the fixed fee just high enough to allow the licensees to make zero profits:

$$F^{SL*} = \frac{M(k^2 + M\gamma + kM\phi)(2k(a_B + k) - 4\gamma c_A + kM\phi + 2(2\gamma + k\phi)a_A)^2}{16n^2\gamma(k^2 + 2M\gamma + 2kM\phi)^2}$$

The optimal fixed fee decreases with the number of licensees (n) and approaches zero when the number of licensees is extremely large. Intuitively, when there are more licensees, the competition among them decreases the profits of each licensee and thus firm A can collect less from each of them.

From the expression of L^{SL*}, it is clear that when $n = 1$, $L^{SL*} < 0$ and when $n > 1$, $L^{SL*} > 0$. When $n = 1$, the variable licensing fee is negative, which means firm A subsidizes the only licensee at a per unit basis but charges a positive fixed fee. The purpose of negative royalty rate is to assure firm B that the only licensee will produce more than monopoly output. The licensing arrangement here becomes a credible commitment to the future output. By licensing, firm A exploits the complementary effects and encourages higher output from B.

By substituting the resulting equilibrium licensing fees and quantities back into (4.21), we have the equilibrium profit of firm A:

$$\pi_A^{SL*} = \frac{M(2k(a_B + k) + 4\gamma(a_A - c_A) + k\phi(2a_A + M))^2}{32\gamma(k^2 + 2M\gamma + 2kM\phi)} \tag{4.24}$$

Lemma 1 *For firm A under complementary effects providing a single version of the product, licensing may deliver a better profit than selling directly.*

The above conclusion is arrived by comparing the equilibrium profits under licensing (see equation (4.24)) with the equilibrium profits under direct selling (see equation (4.12)) when firm A only provides one version of the product.

The Lemma confirms that if firm A could squeeze every penny out of the licensees, a licensing structure as proposed above would lead to a better profit for firm A than selling directly. In reality, firm A may have to give up some profit margin to the licensees. But if licensees only take away a small portion of the profits, firm A still has a chance to earn a profit better than selling directly.

Note that hybrid licensing can only be as good as direct selling for firm A if her product is complementary effect free. So for firms who are not indirectly interacting with a related industry, they may choose to produce and sell the product directly to the end market instead of dealing with the licensing contracts.

4.5.2 When Both Versions Are Provided

As shown above, hybrid licensing can balance the complementary effects and competition. If double-marginalization can be well controlled, firm A can make more profit than selling the product directly to the market. Now, we want to verify that hybrid licensing can play a similar role when there are more than one versions available.

Firm A licenses the technologies of high end and low end products to other manufacturing firms. Assume each licensee only produces one quality, either high or low. n_H is the number of licensees who make the high end version of product A, and n_L is the number of licensees who produce the low end product. We treat n_H and n_L as continuous numbers.

We adopt a three stage game and use backward induction to derive the subgame perfect equilibrium. Firm A moves first by specifying the respective fixed license fees and royalty rates for both versions of product A. F_H and L_H are the fixed fee and royalty rate for the high end version. F_L and L_L are the fixed fee and royalty rate for the low end version. In response to licensee fees, licensees enter the industry until each one earns his indifference profits, which we have assumed to be zero. Finally, all the licensees and producer B simultaneously determine the output quantities. Let q_{AHi} denote the output of high end licensee $i = 1, ..., n_H$ and q_{ALi} be the output of low end licensee $i = 1, .., n_L$. Q_{AHT} is the total output of high end and Q_{ALT} is the total output of the low end. y denotes the output of the complementary producer. At the final stage of the game, where quantities of output are being determined, the profit functions for high and low end licensees are as follows:

$$\begin{aligned}
&\pi^{BL}_{AHi}(q_{AH1}, ..., q_{AHn_H}, q_{AL1}, ..., q_{ALn_L}, y, L_H, L_L, F_H, F_L) \\
=&q_{AHi}(p_{AH}(Q_{AHT}, Q_{ALT}, y) - L_H) - F_H \qquad (4.25) \\
&\pi^{BL}_{ALi}(q_{AH1}, ..., q_{AHn_H}, q_{AL1}, ..., q_{ALn_L}, y, L_H, L_L, F_H, F_L) \\
=&q_{ALi}(p_{AL}(Q_{AHT}, Q_{ALT}, y) - L_L) - F_L \qquad (4.26)
\end{aligned}$$

where $p_{AH}(Q_{AHT}, Q_{ALT}, y)$ and $p_{AL}(Q_{AHT}, Q_{ALT}, y)$ are from equations (4.5) and (4.6).

The profit functions for firm B and firm A are:

$$\begin{aligned}
&\pi^{BL}_{B}(q_{AH1}, ..., q_{AHn_H}, q_{AL1}, ..., q_{ALn_L}, y, L_H, L_L, F_H, F_L) \\
=&y\, p_B(Q_{AHT}, Q_{ALT}, y) \qquad (4.27) \\
&\pi^{BL}_{A}(q_{AH1}, ..., q_{AHn_H}, q_{AL1}, ..., q_{ALn_L}, y, L_H, L_L, F_H, F_L) \\
=&n_H F_H + n_L F_L + L_H Q_{AHT} + L_L Q_{ALT} \qquad (4.28)
\end{aligned}$$

High end and low end licensees of product A and firm B set their own output to maximize their respective profits. By applying first order conditions to (4.25), (4.26), and (4.27), we can determine the best response functions in terms of output quantities of producer B and high end and low end licensees: $y^{BL}(L_H, L_L, n_H, n_L)$, $Q^{BL}_{AHi}(L_H, L_L, n_H, n_L)$, and $Q^{BL}_{ALi}(L_H, L_L, n_H, n_L)$ (The expressions are too long and are omitted here.).

By substituting these best response functions into licensees' profit functions (4.25) and (4.26), we can derive the equilibrium fixed fees as functions of royalty rates and the numbers of high and low end licensees: $F^{BL}_H(L_H, L_L, n_H, n_L)$ and $F^{BL}_L(L_H, L_L, n_H, n_L)$.

To obtain firm A's profit function when she considers the licensing arrangement, we substitute $y^{BL}(L_H, L_L, n_H, n_L)$, $Q^{BL}_{AHT}(L_H, L_L, n_H, n_L)$, $Q^{BL}_{ALT}(L_H, L_L, n_H, n_L)$, $F^{BL}_H(L_H, L_L, n_H, n_L)$, and $F^{BL}_L(L_H, L_L, n_H, n_L)$ into firm A's profit function as defined in equation (4.28). Now, firm A's profit depends only on L_H, L_L, n_H, and n_L. By applying first order conditions to firm A's profit function with respect to L_H and L_L, we derive the optimal licensing arrangement given the numbers of high and low end licensees. It can be shown that any numbers of high and low end licensees (n_H and n_L) combined with the following licensing fees can bring the optimal licensing proceeds to firm A.

$$
\begin{aligned}
L_L^{BL*} =& \frac{(k^2(n_L-2)+2Ms_L\gamma(n_L-1)+2kM\phi(n_L-1))(2k(a_B+k)-4\gamma c_A+kM\phi)}{8n_L\gamma(k^2+2Ms_L\gamma+2kM\phi)} \\
&+ \frac{a_A}{4Mn_L\gamma(k^2+2Ms_L\gamma+2kM\phi)}\big(2k^4+4M^2n_Ls_L^2\gamma^2+k^3M(4+n_L)\phi+ \\
&2kM^2s_L\gamma\phi(1+3n_L)+2k^2M(1+n_L)(s_L\gamma+M\phi^2\,\big) \\
L_H^{BL*} =& \frac{1}{8Mn_H\gamma}\big(4a_A(M\gamma(n_H-1)-k^2)-2a_AkM\phi(n_H-2)+ \\
&+ Mn_H(2k(a_B+k)-4c_A\gamma+kM\phi)\,\big) \\
F_L^{BL*} =& \frac{(k^2+Ms_L\gamma+kM\phi)(-2a_Ak(k+M\phi)+M(2k(a_B+k)-4c_A\gamma+kM\phi)^2}{16Mn_L^2\gamma(k^2+2Ms_L\gamma+2kM\phi)^2} \\
F_H^{BL*} =& \frac{a_A^2(k^2+M\gamma+kM\phi)}{4Mn_H^2\gamma}
\end{aligned}
$$

Note that the licensing fees for the high end licensees, L_H^{BL*} and F_H^{BL*}, are both a function of the number of high end licensees (n_H), but independent of the number of low end licensees (n_L). We observe the similar pattern in the licensing fees for the low end licensees. Intuitively, the optimal high end output and the optimal low end output are fixed numbers. Firm A only needs to adjust L_H^{BL*} and F_H^{BL*} according to the number of high end licensees to induce the optimal total high end output and grab the profits from them.

Fixed fees F_H^{BL*} and F_L^{BL*} are decreasing in n_H and n_L, respectively. The royalty rates L_H^{BL*} and L_L^{BL*} increases with n_H and n_L, respectively.

Substituting the optimal licensing fees into licensees and firm B's best response functions, $y^{BL}(L_H, L_L, n_H, n_L)$, $Q_{AHT}^{BL}(L_H, L_L, n_H, n_L)$, and $Q_{ALT}^{BL}(L_H, L_L, n_H, n_L)$, we can have the equilibrium output quantities when firm A licenses both technologies to other firms.

$$Q_{AHi}^{BL*} = \frac{a_A}{2n_H} \tag{4.29}$$

$$Q_{ALi}^{BL*} = \frac{M\left(2k\left(a_B + k\right) - 4\gamma c_A + kM\phi\right) - 2a_A k(k + M\phi)}{4n_L\left(k^2 + 2Ms_L\gamma + 2kM\phi\right)} \tag{4.30}$$

$$y^{BL*} = \frac{2a_A k + 8a_B M + kM - 4M\phi(M - 2a_A)}{16} + \frac{k(k(-2a_A k + 4a_B M + 3kM) - 2M\gamma(4c_A + s_L(M - 2a_A)))}{16(k^2 + 2Ms_L\gamma + 2kM\phi)} \tag{4.31}$$

Note that only when $M\left(2k\left(a_B + k\right) - 4\gamma c_A + kM\phi\right) - 2a_A k(k + M\phi) > 0$, the anticipated low end output is positive. Similar to the analysis in the direct selling case, it is easy to verify that $M\left(2k\left(a_B + k\right) - 4\gamma c_A + kM\phi\right) - 2a_A k(k + M\phi)$ is continuous in k and negative when $k = 0$. It equals zero at some $k_1' < 0$ and $k_2' > 0$. As k is positive by our assumption, it follows that only when $k > k_2' > 0$, both versions are available to the market. We denote this threshold of k beyond which low end product would be available under hybrid licensing as K_L.

Similar to the direct selling case, it is straightforward to see that when $k > K_L$, the total output of high and low end licensees is larger than the output when only the high end product is available through licensing.

Proposition 12 *a).There is a threshold K_L of the complementary effect parameter k, above which an innovator who licenses will introduce a low end product to the market through licensees.*

b). $K_L < K_D$

Part a) follows from the fact that only when $k > K_L$, firm A would have incentive to induce positive output of low end product from licensees. The Proposition shows that when the complementary effects are strong enough, firm A would offer both versions to the market through licensees. By introducing a low end version, the total number of customers who buy product A would increase and thus would encourage the manufacturer of product B to increase the output as well. As a result, customers may be willing to pay more for product A as product A and B are complements. If such complementary positive feedback is strong enough, firm A would benefit from introducing the low end.

Recall that for firm A as a direct seller to introduce the low end version, we must have $M\left(2k\left(a_B+k\right)-4\gamma c_A+kM\phi\right)-a_Ak(3k+2M\phi)>0$. On the other hand, only when $M\left(2k\left(a_B+k\right)-4\gamma c_A+kM\phi\right)-2a_Ak(k+M\phi)>0$, firm A that licenses the technology would introduce the low end version to the market through licensees. As $M\left(2k\left(a_B+k\right)-4\gamma c_A+kM\phi\right)-2a_Ak(k+M\phi)>M\left(2k\left(a_B+k\right)-4\gamma c_A+kM\phi\right)-a_Ak(3k+2M\phi)$, we have $K_L<K_D$.

When firm A directly sells the product, only when the complementary effects are relatively strong, i.e., $k>K_D$, a low end would be profitably introduced to the market. However, if firm A licenses instead of selling directly, the threshold strength of the complementary effects is lower. This is an example of strategic complementarity, i.e., the implementation of one strategy encourages the use of the other. When firm A licenses, the expected future output is higher, thus encourages the output from the complementor. When firm A expands the product line, the future sales is higher and thus can induce more output from the complementary producer. However, due to cannibalization, only when the complementary effects are strong, the expansion of the product line can be justified. However, under technology licensing, firm A can better leverage the complementary effects and thus a smaller level of the effects can provide enough incentive for firm A to introduce the low end version.

Corollary 5 *When product A is independent of other product, i.e., $k = 0$, firm A only provides the high end version to the market through licensing.*

The Corollary follows from the fact that when $k = 0$, the equilibrium low end quantity is negative. The Corollary confirms that firm A behaves differently when her product is independent of other markets and when she interacts with a complementary producer. Firm A that licenses the technology under complementarity may provider a broader product line than she would if her product was independent of other markets.

Corollary 6 *If firm A licenses the technology that interacts with a complementary product, the quality level of the low end has no effect on her decision regarding whether or not to include the low end into the product line.*

The Corollary is true because the term that determines the value of the threshold K_L, $M\left(2k\left(a_B + k\right) - 4\gamma c_A + kM\phi\right) - 2a_Ak(k + M\phi)$, does not contain s_L. From (4.30), although the low end output Q_{ATL}^{BL*} decreases with s_L, s_L does not affect the decision regarding whether or not to introduce the low end product.

By substituting the resulting equilibrium output quantities back into (4.28), we have firm A's equilibrium profits π_A^{BL*} when $k > K_L$. Note that any number of low end and high end licensees combined with above optimal licensing fees can bring firm A such optimal profits.

Proposition 13 *For any value of $k > 0$, hybrid licensing may enable firm A to earn a better profit than selling directly to market by herself.*

Proof:

a). When $k < K_L < K_D$, firm A would provide only the high end version under both direct selling and licensing. By Lemma 1, we know licensing is better.

b). When $K_L < k < K_D$, firm A would provide only the high end version when she direct sells and both versions when she licenses. By optimization, the licensing proceeds from licensing both versions are higher than those from licensing only the high version. By Lemma 1, the profits from licensing are higher than direct selling when only high end version is provided.

c). When $K_L < K_D < k$, firm A provides both versions when direct selling and licensing. By comparing the equilibrium profits of A under two situations, we arrive at the conclusion that licensing is better. $\Diamond$

The proposition shows that a hybrid licensing arrangement can bring a higher profit as the royalty rate scheme can induce an optimal output quantities for both high and low end version of product A. In the context of a differentiated product A, these quantities not only best balance the trade-off between the complementary effects and competition, but also coordinate the interactions between high end and low end markets. Thus for any numbers of high end and low end licensees, with an appropriate royalty rate to induce the right quantity and a fixed licensing fee to squeeze the licensees, firm A can deliver a better than direct selling outcome by using a hybrid licensing scheme.

Corollary 7 *If firm A that licenses can choose the quality of the low end good, she would set that as low as possible under hybrid licensing.*

The above Corollary shows that firm A would set the quality of the low end version of product A to the lowest possible level. In general, such product line decisions lead to the maximum total sales of product A, but generate the greatest cannibalization between high and low end product. However, with the hybrid licensing arrangement cushioning the cannibalization and leveraging the complementary effects, it is beneficial for firm A to adopt such an extreme strategy. Note that due to the complementary effects, although low end version may be worth nothing in isolation to all consumers, the product is marketed at a positive price. At least some consumers are willing to pay a price for the low end version so as to have the use of both product A and B.

4.6 Concluding Remarks

In this research, we study an innovator's product line strategy when her product interacts with a complementary industry. The innovator can directly serve the market or license the technology to other manufacturing companies.

While a broader product line can serve as an assurance to the complementary producer about higher future output levels, it certainly creates the cannibalization between high and low end products. Under the extreme situation where the production costs are the same for both versions, where the high end enjoys a much higher margin, the cannibalization can present a major threat to a company's effort to maximize the profits. That's the reason why companies whose products are independent of other markets would not benefit from introducing a low end product if the low end product can not be produced at a lower cost.

However, when the product is involved in a complementary relationship with another market and when such connection is strong enough, companies have incentives to expand the product line even when the low end version costs the same as the high end. A broader product line is a credible commitment to higher future output and thus encourages higher output from the complementary producer, which in turn would boost the demand for the product.

Companies whose products have complements may introduce a low end version when they serve the market directly and when they license the technology as long as the complementary interactions are above some threshold levels. However, the threshold under licensing is smaller than that under direct selling. As licensing can constitute a credible commitment to higher output level, it can help offset the cannibalization and thus relatively moderate complementary effects can justify introduction of a low end version. Thus licensing and providing a broader product line can serve the same purpose and are strategic complements to each other.

Whenever complementary effects are present, hybrid licensing may bring better profits to companies than direct selling. The royalty rate in licensing can balance among the complementary effects, competition, and cannibalization. It results in larger total gains across the industry. So long as the licensees do not take away a big portion of the profits, the companies have a chance to obtain a better than direct selling net gain under licensing.

Finally, the quality of the low end has no effect on company's product line decision although it certainly influences the output of the low end if the low end is provided. In addition, under the complementary effects, companies prefer to have a low end with the lowest possible quality.

There are several limitations of our research. First of all, different qualities may be produced at different costs. Our model assume constant variable costs across qualities. It is of interest to see to what extent the firms will extend their product line based on different cost-quality functions. There are certainly other ways to trade off between complementary effects and competition and induce the optimal outputs. We may want to identify those schemes and compare them with the industry practices. Finally, empirical studies about the output and product line decisions of companies interacting with a complementary industry are also desired.

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